

ADVANCED SUBSIDIARY GCE
MATHEMATICS (MEI)
Decision Mathematics 1

4771

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- Printed Answer Book
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Wednesday 17 June 2009
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

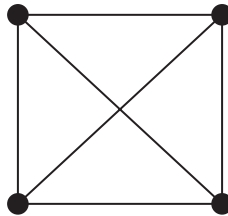
Answer all questions in the printed answer book provided.

Section A (24 marks)

- 1 The numbers on opposite faces of the die shown (a standard die) add up to 7. The adjacency graph for the die is a graph which has vertices representing faces. In the adjacency graph two vertices are joined with an arc if they share an edge of the die. For example, vertices 2 and 6 are joined by an arc because they share an edge of the die.



- (i) List the pairs of numbers which are opposite each other. [1]
- (ii) Draw the adjacency graph. [5]
- (iii) Identify and sketch a solid which has the following adjacency graph. [2]



- 2 In this question $\text{INT}(m)$ means the integer part of m . Thus $\text{INT}(3.5) = 3$ and $\text{INT}(4) = 4$.

A game for two players starts with a number, n , of counters. Players alternately pick up a number of counters, **at least 1** and **not more than half** of those left. The player forced to pick up the last counter is the loser. Arif programs his computer to play the game, using the rule “pick up $\text{INT}(\text{half of the remaining counters})$, or the last counter if forced”.

- (i) You are to play against Arif’s computer with $n = 5$ and with Arif’s computer going first. What happens at each turn? [3]
- (ii) You are to play against Arif’s computer with $n = 6$ and with Arif’s computer going first. What happens at each turn? [2]
- (iii) Now play against Arif’s computer with $n = 7$ and with Arif’s computer going first. Describe what happens. [3]

3 Consider the following linear programming problem:

Maximise $3x + 4y$

subject to $2x + 5y \leq 60$
 $x + 2y \leq 25$
 $x + y \leq 18$

- (i) Graph the inequalities and hence solve the LP. [6]
- (ii) The right-hand side of the second inequality is increased from 25. At what new value will this inequality become redundant? [2]

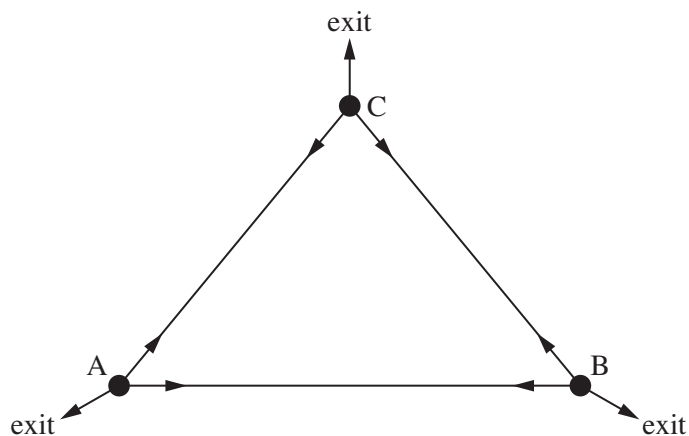
Section B (48 marks)

- 4 The diagram represents a very simple maze with two vertices, A and B. At each vertex a rat either exits the maze or runs to the other vertex, each with probability 0.5. The rat starts at vertex A.



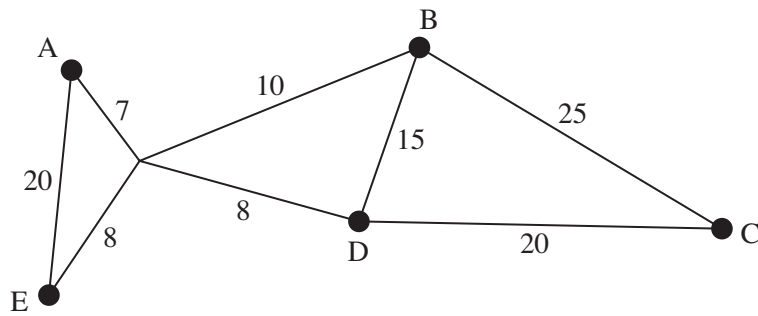
- (i) Describe how to use 1-digit random numbers to simulate this situation. [2]
- (ii) Use the random digits provided in your answer book to run 10 simulations, each starting at vertex A. Hence estimate the probability of the rat exiting at each vertex, and calculate the mean number of times it runs between vertices before exiting. [5]

The second diagram represents a maze with three vertices, A, B and C. At each vertex there are three possibilities, and the rat chooses one, each with probability $\frac{1}{3}$. The rat starts at vertex A.



- (iii) Describe how to use 1-digit random numbers to simulate this situation. [4]
- (iv) Use the random digits provided in your answer book to run 10 simulations, each starting at vertex A. Hence estimate the probability of the rat exiting at each vertex. [5]

- 5 The diagram represents canals connecting five cities. Canal lengths (shown on the arcs) are in km.

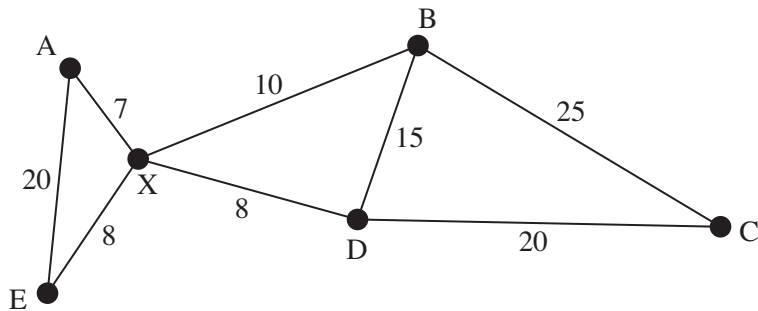


- (i) Draw a network in your answer book with nodes representing the five cities and arcs representing direct canal connections, i.e. canal connections which do not involve passing through another city. [3]

The company operating the canal system wishes to close some canals to save money, whilst preserving the connectivity.

- (ii) Starting at A, use Prim's algorithm on your answer to part (i) to find a minimum connector for the network. Give the order in which you include arcs. Draw your minimum connector and give its total length. [5]

Consider the original network together with an extra vertex, X, at the junction of four arcs.



- (iii) Draw the minimum connector which results from applying Prim's algorithm, starting at A, to this network. Give the length of that minimum connector. Hence advise the company on which canals to close. [7]
- (iv) Give a reason why the company might face objections to such closures. [1]

Question 6 is printed overleaf.

- 6 Joan and Keith have to clear and tidy their garden. The table shows the jobs that have to be completed, their durations and their precedences.

	Jobs	Duration (mins)	Immediate predecessors
A	prune bushes	100	–
B	weed borders	60	A
C	cut hedges	150	–
D	hoe vegetable patch	60	–
E	mow lawns	40	B
F	edge lawns	20	D, E
G	clean up cuttings	30	B, C
H	clean tools	10	F, G

- (i) Draw an activity on arc network for these activities. [4]
- (ii) Mark on your diagram the early time and the late time for each event. Give the minimum completion time and the critical activities. [6]
- (iii) Each job is to be done by one person only. Joan and Keith are equally able to do all jobs. Draw a cascade chart indicating how to organise the jobs so that Joan and Keith can complete the project in the least time. Give that least time and explain why the minimum project completion time is shorter. [6]

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**ADVANCED SUBSIDIARY GCE
MATHEMATICS (MEI)**

Decision Mathematics 1

PRINTED ANSWER BOOK

4771

**Wednesday 17 June 2009
Morning**

Duration: 1 hour 30 minutes



Candidate Forename		Candidate Surname	
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Centre Number						Candidate Number				
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INSTRUCTIONS TO CANDIDATES

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- Use black ink. Pencil may be used for graphs and diagrams only.
- Write your answers in the spaces provided on the answer book. If extra paper is required use a 4 page answer booklet making sure that you label your work clearly. Attach any extra answer booklets to this Printed Answer Book.

INFORMATION FOR CANDIDATES

- This document consists of **12** pages. Any blank pages are indicated.

1 (i)

(ii)

Vertex 6 ●

● Vertex 1

Vertex 5 ●

● Vertex 2

Vertex 4 ●

● Vertex 3

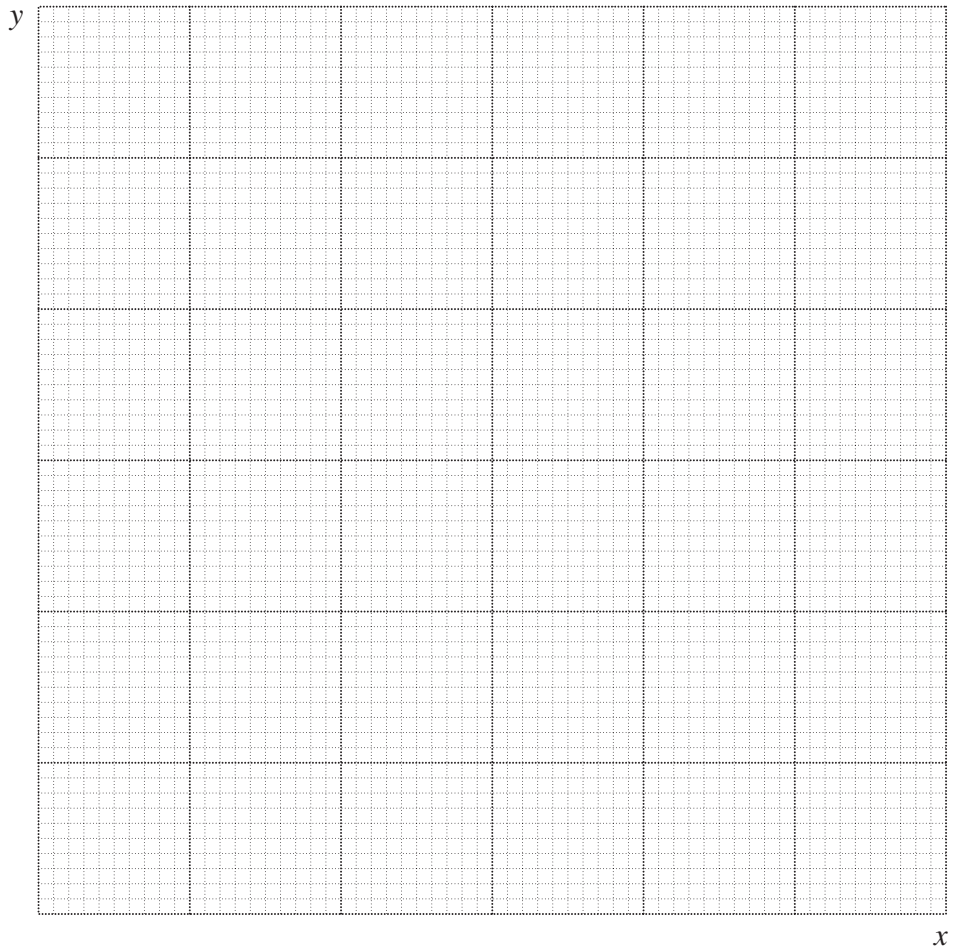
(iii)

2 (i)

(ii)

(iii)

3 (i)

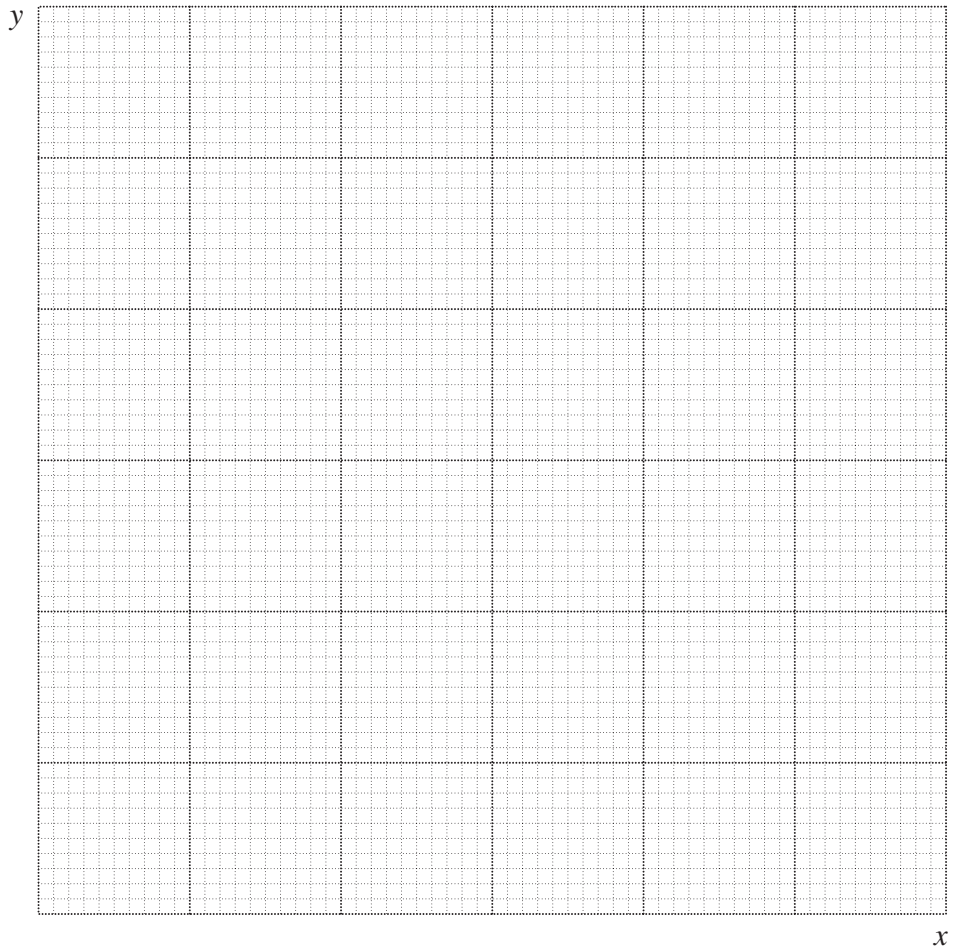


Solution:

(ii)

3 (i)

SPARE COPY OF GRAPH PAPER FOR QUESTION 3(i)



Solution:

4 (i)

(ii) Random digits

run 1	1	5	6	4	9	4	6	0	0	9
run 2	8	7	8	6	6	3	2	9	7	1
run 3	2	1	9	4	9	1	8	2	5	1
run 4	6	8	7	9	4	6	0	6	6	8
run 5	6	0	5	7	7	1	7	8	5	1
run 6	6	7	7	9	7	2	2	7	4	7
run 7	5	7	7	3	2	7	1	1	5	5
run 8	0	7	3	9	6	8	0	9	2	3
run 9	9	4	2	9	2	3	2	6	0	1
run 10	2	8	5	8	6	9	1	4	8	3

Simulation runs

run 1	A
run 2	A
run 3	A
run 4	A
run 5	A
run 6	A
run 7	A
run 8	A
run 9	A
run 10	A

Probability of exiting at A: Probability of exiting at B:

Mean number of runs between vertices:

(iii)

(iv) Random digits

run 1	4	6	5	6	1	1	6	5	2	2
run 2	7	4	1	2	5	2	4	8	8	6
run 3	1	5	8	8	2	7	8	8	9	3
run 4	4	4	8	9	4	1	4	9	1	0
run 5	2	0	3	3	1	5	7	5	1	6
run 6	6	5	3	0	4	5	8	2	9	2
run 7	2	3	5	8	2	3	7	4	7	6
run 8	3	5	1	7	6	9	4	0	4	6
run 9	0	9	1	6	4	2	2	4	5	3
run 10	0	5	0	6	9	1	3	6	0	0

Simulation runs

run 1	A
run 2	A
run 3	A
run 4	A
run 5	A
run 6	A
run 7	A
run 8	A
run 9	A
run 10	A

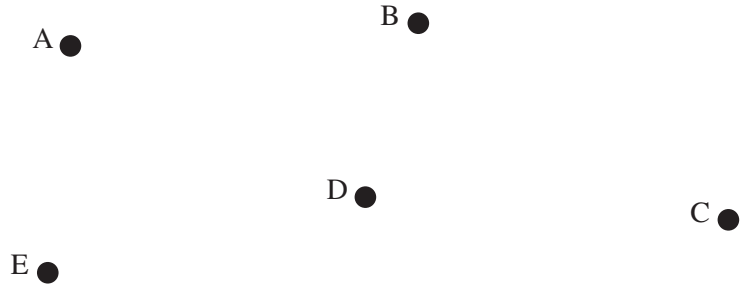
Probability of exiting at A:

Probability of exiting at B:

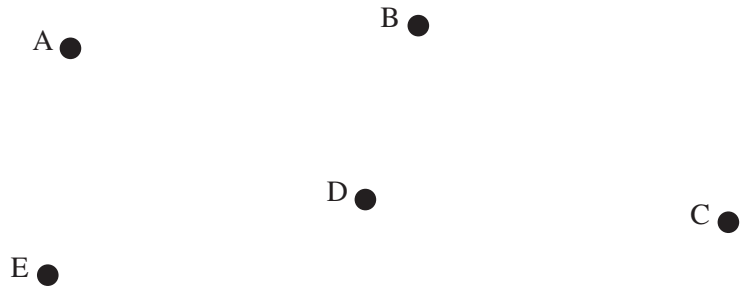
Probability of exiting at C:

8

5 (i)



(ii)

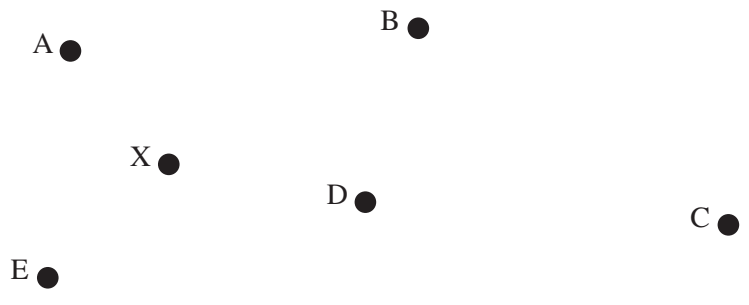


Order of inclusion:

Total length:

9

(iii)



Total length:

Advice:

(iv)

6 (i)&(ii)

Minimum completion time:

Critical activities:

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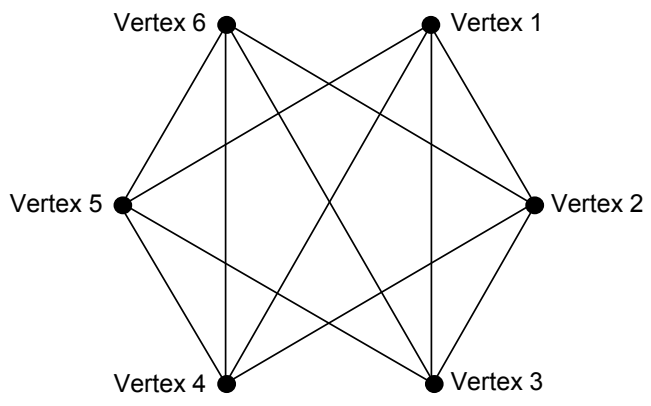
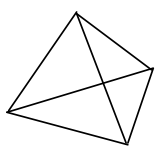
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4771 Decision Mathematics 1

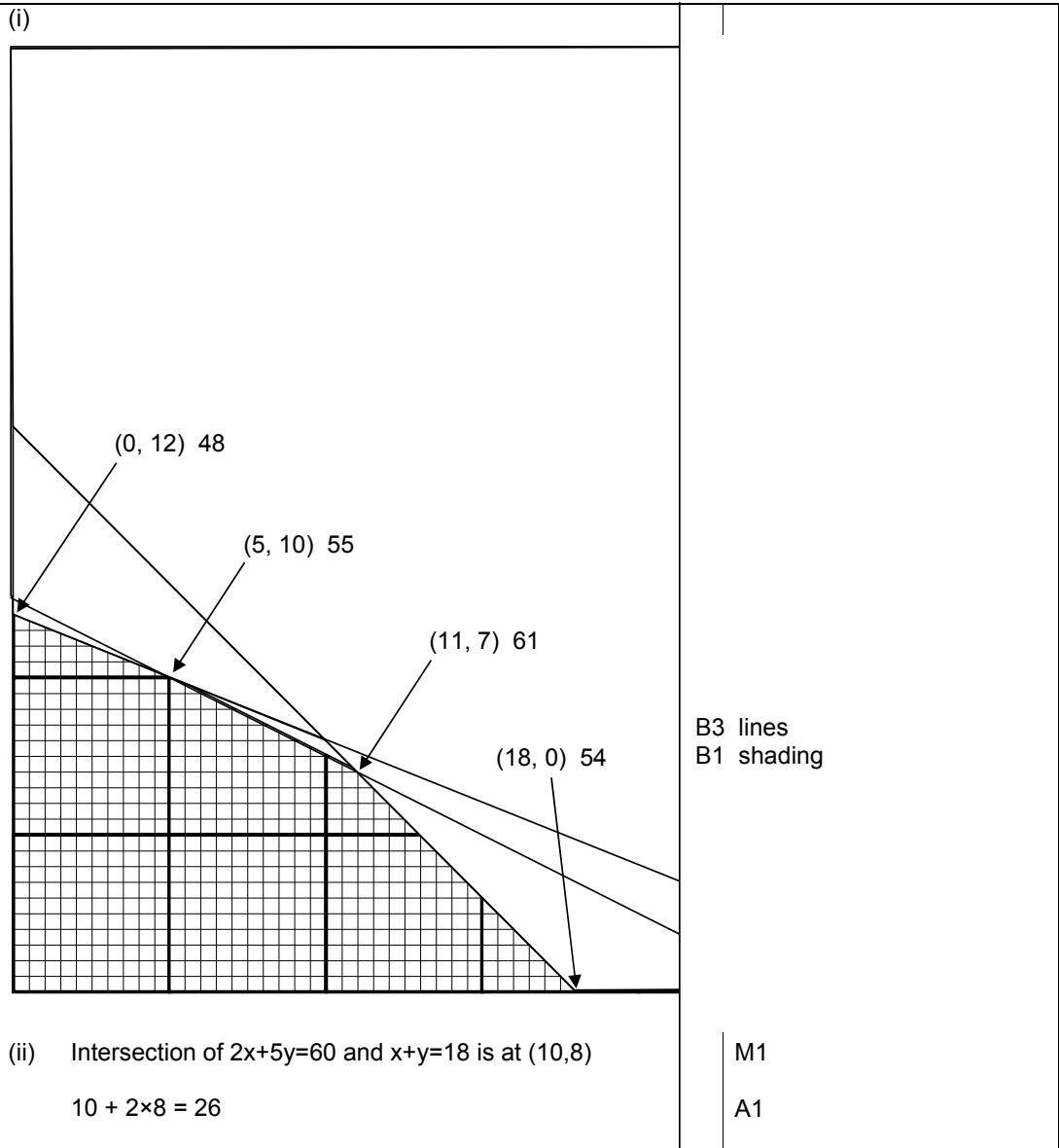
Question 1

<p>(i) 1 and 6, 2 and 5, 3 and 4</p> <p>(ii)</p>  <p>(iii)</p> 	<p>B1</p> <p>M1 10 to 14 edges A4 (-1 each edge error)</p> <p>B1 identification B1 sketch</p>
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Question 2.

<p>(i) A's c takes 2, leaving 3. You have to take 1. A's c takes one and you lose.</p> <p>(ii) A's c takes 3 leaving 3. Then as above.</p> <p>(iii) A's c takes 3 leaving 4. You can then take 1, leading to a win.</p>	<p>M1 A1 A1</p> <p>M1 A1</p> <p>M1 A1 A1</p>
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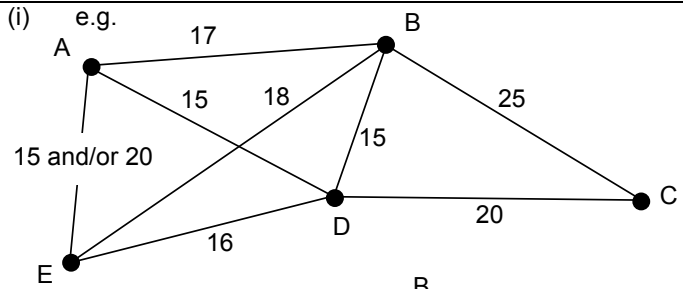
Question 3.



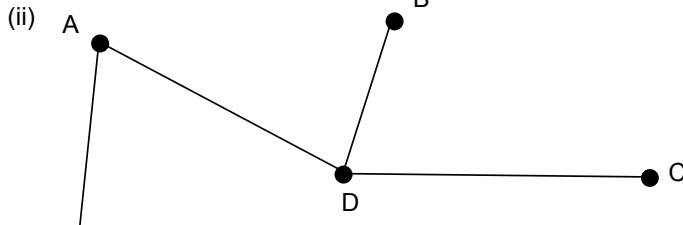
Question 4.

(i)	e.g. 0–4 exit	5–9 other vertex							B1 B1
(ii)	e.g.	1 A ExA							M1 process with exits
		2 A B A B A B ExB							A1
		3 A ExA							
		4 A B A B A ExA							
		5 A B ExB							
		6 A B A B A B ExB							
		7 A B A B ExB							
		8 A ExA							
		9 A B ExB							
		10 A ExA							
	0.5, 0.5, 1.9	(Theoretical answers: 2/3, 1/3, 2)							B1 probabilities
		(Gambler's ruin)							M1 duration
									A1
(iii)	e.g. 0–2 exit	3–5 next vertex in cycle							M1 ignore
	6–8 other vertex	9–ignore and re-draw							DM1 conditionality
									A1 equal prob
									A1 efficient
(iv)	e.g.	1 A B A B A ExA							M1
		2 A C A ExA							A2
		3 A ExA							
		4 A B C B C ExC							
		5 A ExA							
		6 A C A B ExB							
		7 A ExA							
		8 A B C ExC							
		9 A ExA							
		10 A ExA							
	0.7, 0.1, 0.2	(Theoretical probs are 0.5, 0.25, 0.25)							M1
		(Markov chain)							A1

Question 5.



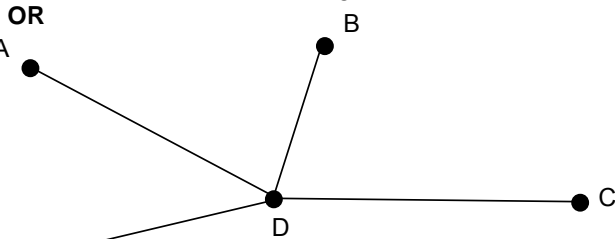
M1
A1 connectivity
A1 lengths



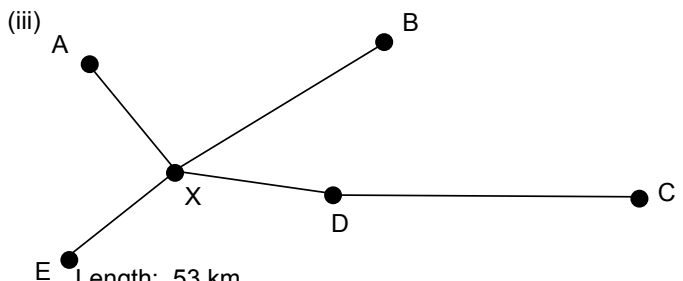
M1 connected tree
A2 (-1 each error)

Order: AE; AD; DB; DC or
AD; AE; DB; DC or
AD; DB; AE; DC Length: 65 km

A1 B1



Order: AD; DB; DE; DC Length: 66 km



M1 connected tree
A2 (-1 each error)

Length: 53 km
Advice: Close BC, AE and BD

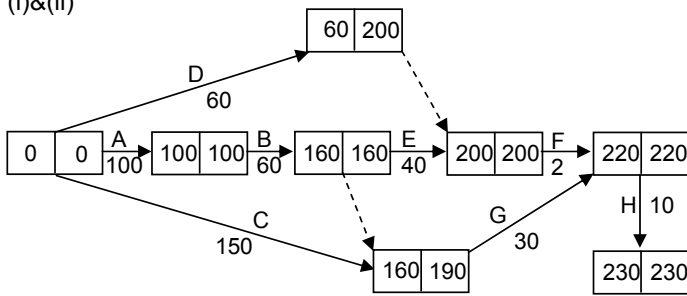
B1
B3

(iv) facility (e.g. anglers)
distances (e.g. B to C)

B1

Question 6.

(i)&(ii)



time – 230 minutes
critical – A; B; E; F; H

(iii) e.g.



Least time = 240 mins

Minimum project completion times assumes no resource constraints.

M1 sca (activity on arc)
A1 single start & end
A1 dummy
A1 rest

M1 forward pass
A1
M1 backward pass
A1

B1
B1 cao

M1 cascade
A2

B1 Joan/Keith

B1

B1

4771 Decision Mathematics 1

General Comments

This proved an accessible paper with a number of good candidates scoring highly.

A comment must be made about the marking process. Candidates' scripts are now scanned in, and the module is marked online. Most of the scanning is in black and white, so colour should not be used. Correcting work by rubbing out was not always done as well as it could be, sometimes leaving ambiguous answers. Correction fluid does not work well with scanning, and should not be used.

There was some evidence that a minority of candidates had difficulty finishing the paper in the time allowed, but in virtually all cases this was due to spending time unnecessarily on question 2 (verbose writing) and question 4 (using all 100 random numbers – twice). Correct answers required the use of about 20% of the numbers.

Comments on Individual Questions

1) **Graphs**

Most were able to collect the marks from parts (i) and (ii).

In part (iii) there was some misnaming of a tetrahedron, but most candidates were able to describe the shape adequately enough to gain the mark. Fewer were able to sketch it.

2) **Algorithms**

This question was answered well in all parts by the majority of candidates, although a small minority failed to grasp the essential point that the computer's algorithm was just that – there was no element of choice.

Most problems that occurred were in part (iii) when some candidates forgot that, although the computer did not have a choice, the other player did, and would exercise that choice to choose a strategy which would lead to a win.

Some candidates used efficient terminology, but others wasted much time writing copious paragraphs when much less wordy answers were preferable. In some cases this seemed to lead to the candidate being rushed to answer the final question.

3) **Linear Programming**

In part (i) most candidates were able to draw the inequalities, although a surprising number failed to identify the feasible region by appropriate shading. For full marks in identifying the optimal point, it was necessary to show working. This could be done either by drawing a profit line on the diagram, or by calculating the value of the objective function at each vertex on the edge of the feasible region. Many candidates ignored the point (5, 10), and so failed to gain maximum marks on this part.

Part (ii) was found to be difficult. Candidates needed to find the point (10, 8), to realise that the inequality would become redundant at that point, substitute the values into it and hence get 26. Again, some indication of methodology was required. Answers of 26 with no working were not adequate.

4) **Simulation**

When candidates interpreted this question correctly they answered it very well. In part (i) the creation of a simulation rule without omitting any numbers was handled well. The interpretation of the process in part (ii), in which rats ran or exited, was handled less well.

In part (iii) the process was widened to rats going to one of two alternatives or exiting. Most candidates realised a digit had to be omitted to create the simulation rule, but only some realised that the rule needed in some way to take account of which vertex was current. This led to many and varied answers for the simulation in part (iv).

5) **Networks**

Some candidates had problems interpreting what parts (i) and (ii) were asking, and hence dropped marks. The question was designed to allow those candidates to recover in parts (iii) and (iv), and most candidates scored highly on those parts. In part (i) candidates were required to compute show the network of direct connections, and in part (ii) they then had to find the minimum connector. It was not uncommon to see exactly the same answer given for parts (i) and (ii).

Part (iii) was answered well and in part (iv) most were able to give a plausible real-life reason why the minimum connector was not necessarily the best solution.

6) **CPA**

This question was pleasingly well done. Candidates often scored full marks in parts (i) and (ii) in which they had to construct the network, do a forward and backwards pass, and identify critical activities.

In part (iii) the number of workers was limited to two, and a work schedule had to be produced on a cascade chart. Most candidates failed to identify which tasks were to be undertaken by which worker, and many had schedules in which more than two tasks were being undertaken at the same time.

In the final explanation part, many seemed to regard having two workers in part (iii) as being better than the situation in parts (i) and (ii), whereas those parts assumed no resource constraints.