1)

$$x^2 = 4 \iff x = 2$$

This is false since

$$x^2 = 4 \text{ when } x = -2$$

2) i)

$$z^2 - 4z + 7 = 0$$

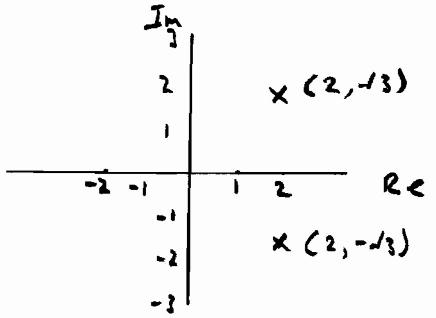
$$z = \frac{4 \pm \sqrt{(-4)^2 - 4\times1\times7}}{2}$$

$$z = \frac{4 \pm \sqrt{-12}}{2}$$

$$z = \frac{4 \pm 2\sqrt{3}\,j}{2}$$

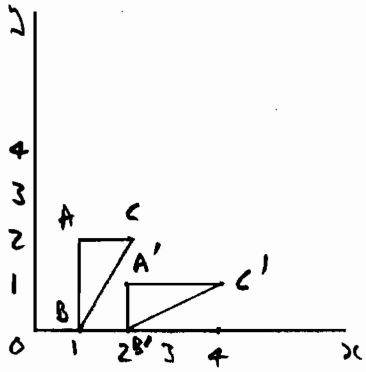
$$z = 2 \pm \sqrt{3}\,j$$

2ii)



3) i)

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} & \begin{pmatrix} 1 & 1 & 2 \\ 2 & 0 & 2 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{pmatrix} 2 & 2 & 4 \\ 1 & 0 & 1 \end{pmatrix} \end{array}$$



3ii) Two way stretch by a scale factor 2 parallel to x axis and scale factor $\frac{1}{2}$ parallel to y axis

4)

$$\sum_{r=1}^{n} r(r^2+1)$$

$$= \sum_{r=1}^{n} (r^3 + r)$$

$$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{2}n(n+1)$$

$$= \frac{1}{2}n(n+1)\left(\frac{1}{2}n(n+1) + 1\right)$$

$$= \frac{1}{4}n(n+1)\left(n(n+1) + 2\right)$$

$$= \frac{1}{4}n(n+1)(n^2 + n + 2)$$

5)

$$2x^3 - 3x^2 + x - 4 = 0$$

Let $w = 2x + 1$

$$\Rightarrow \quad x = \frac{w-1}{2}$$

$$\therefore \quad 2\left(\frac{w-1}{2}\right)^3 - 3\left(\frac{w-1}{2}\right)^2 + \frac{w-1}{2} - 4 = 0$$

$$2\left(\frac{w^3 - 3w^2 + 3w - 1}{8}\right)$$

$$-3\left(\frac{w^2 - 2w - 1}{4}\right) + \frac{w-1}{2} - 4 = 0$$

$$\Rightarrow w^3 - 3w^2 + 3w - 1$$

$$-3w^2 + 6w - 3$$

$$+2w - 2$$

$$-16 = 0$$

5 cont) $w^3 - 6w^2 + 11w - 22 = 0$

has roots $2\alpha+1$, $2\beta+1$, $2\gamma+1$

6) Prove $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$

When $n=1$

$\sum 1^2 = 1$, $\frac{1}{6}(1)(2)(3) = 1$

$\therefore$ true for $n=1$

Assume true for $n=k$ so

$$\sum_{r=1}^{k} r^2 = \frac{1}{6}k(k+1)(2k+1)$$

$$\Rightarrow \sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$$

$$= \frac{1}{6}(k+1)\left[k(2k+1) + 6(k+1)\right]$$

$$= \frac{1}{6}(k+1)\left[2k^2 + k + 6k + 6\right]$$

$$= \frac{1}{6}(k+1)\left[2k^2 + 7k + 6\right]$$

$$= \frac{1}{6}(k+1)\left[(2k+3)(k+2)\right]$$

$$= \frac{1}{6}(k+1)(k+2)(2k+3)$$

$$= \frac{1}{6}(k+1)((k+1)+1)(2(k+1)+1)$$

which is the same formula with $k+1$ replacing k

Hence, if true for $n=k$, formula is also true for $n=k+1$

Since formula is true for $n=1$, by mathematical induction it is true for all positive integers n

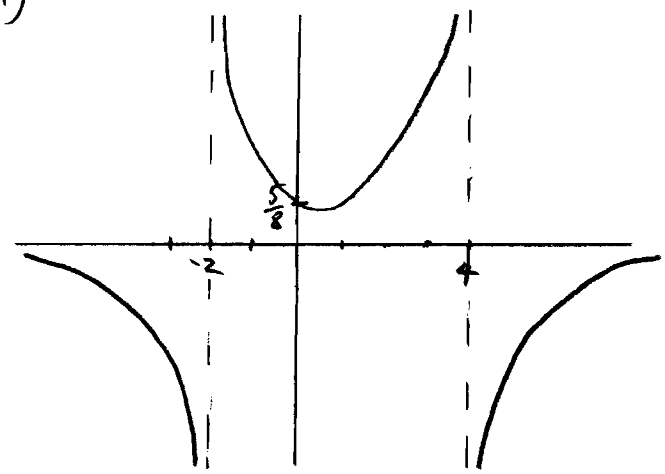
Section B

7) $y = \frac{5}{(x+2)(4-x)}$

i) when $x=0$, $y = \frac{5}{2\times 4} = \frac{5}{8}$

ii) Asymptotes $x=-2$, $x=4$
$y=0$

iii)



When x just <-2 y is $\frac{+}{-\,+} = -$ve

x just >-2 y is $\frac{+}{+\,+} = +$ve

x just <4 y is $\frac{+}{+\,+} = +$ve

x just >4 y is $\frac{+}{+\,-} = -$ve

7iv)

$$\frac{5}{(x+2)(4-x)} = 1$$

$$\Rightarrow 5 = (x+2)(4-x)$$

7 iv) cont)

$$5 = 4x + 8 - x^2 - 2x$$

$$5 = 2x + 8 - x^2$$

$$x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

Solve $\dfrac{5}{(x+2)(4-x)} < 1$

From graph

$-1 < x < 3$

or $x < -2$

or $x > 4$

8)

$$m = -4 + 2j$$

i)

$$\frac{1}{m} = \frac{1}{-4+2j}$$

$$= \frac{-4-2j}{-4-2j} \times \frac{1}{-4+2j}$$

$$= \frac{-4-2j}{16+4}$$

$$= -\frac{1}{5} - \frac{1}{10}j$$

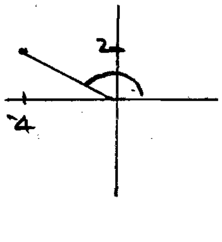
ii)

$$|m| = \sqrt{4^2+2^2} = \sqrt{20}$$

$\arg m =$

$\pi - \tan^{-1}\dfrac{2}{4}$

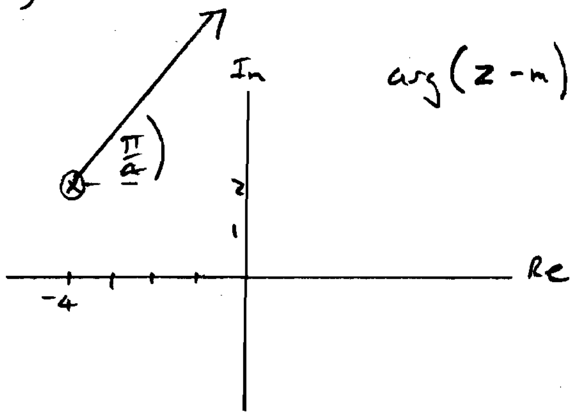
$= 2.68$ radians



$$m = \sqrt{20}\left(\cos 2.68 + j\sin 2.68\right)$$

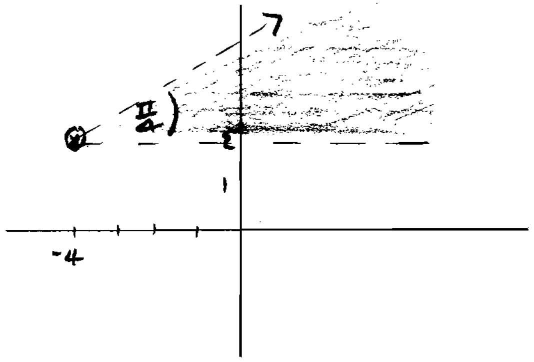
8 iii)

A)



$\arg(z - m) = \frac{\pi}{4}$

B)



Shaded area represents

$$0 < \arg(z - m) < \frac{\pi}{4}$$

9) i)

$$\underline{M} = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix} \quad \underline{N} = \begin{pmatrix} 1 & -3 \\ 1 & 4 \end{pmatrix}$$

$$\underline{M}^{-1} = \frac{1}{3}\begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} \\ 0 & 1 \end{pmatrix}$$

$$N^{-1} = \frac{1}{4--3}\begin{pmatrix} 4 & 3 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{4}{7} & \frac{3}{7} \\ -\frac{1}{7} & \frac{1}{7} \end{pmatrix}$$

9ii) $\underline{MN} = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ 1 & 4 \end{pmatrix}$

$= \begin{pmatrix} 5 & -1 \\ 1 & 4 \end{pmatrix}$

$(\underline{MN})^{-1} = \frac{1}{20+1}\begin{pmatrix} 4 & 1 \\ -1 & 5 \end{pmatrix}$

$= \begin{pmatrix} \frac{4}{21} & \frac{1}{21} \\ -\frac{1}{21} & \frac{5}{21} \end{pmatrix}$

$\underline{N}^{-1}\underline{M}^{-1} = \begin{pmatrix} \frac{4}{7} & \frac{3}{7} \\ -\frac{1}{7} & \frac{1}{7} \end{pmatrix}\begin{pmatrix} \frac{1}{3} & -\frac{2}{3} \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} \frac{4}{21}+0 & -\frac{8}{21}+\frac{9}{21} \\ -\frac{1}{21}+0 & \frac{2}{21}+\frac{3}{21} \end{pmatrix}$

$= \begin{pmatrix} \frac{4}{21} & \frac{1}{21} \\ -\frac{1}{21} & \frac{5}{21} \end{pmatrix} = (\underline{MN})^{-1}$

9iii) Prove $(\underline{PQ})^{-1} = \underline{Q}^{-1}\underline{P}^{-1}$

$(\underline{PQ})^{-1}\underline{PQ} = \underline{I}$

$(\underline{PQ})^{-1}\underline{PQ}\underline{Q}^{-1} = \underline{I}\underline{Q}^{-1}$

$(\underline{PQ})^{-1}\underline{P}\underline{I} = \underline{I}\underline{Q}^{-1}$

$(\underline{PQ})^{-1}\underline{P} = \underline{Q}^{-1}$

$(\underline{PQ})^{-1}\underline{P}\underline{P}^{-1} = \underline{Q}^{-1}\underline{P}^{-1}$

$(\underline{PQ})^{-1}\underline{I} = \underline{Q}^{-1}\underline{P}^{-1}$

$(\underline{PQ})^{-1} = \underline{Q}^{-1}\underline{P}^{-1}$

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