1) $\underline{A} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ $\underline{B} = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$

i) $\underline{B}\,\underline{A} = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ -4 & 14 \end{pmatrix}$

ii) $\det A = 6 - 0 = 6$

$\det B = 12 - -2 = 14$

Area $= 14 \times 6 \times 3 = 252$ units^2

2)

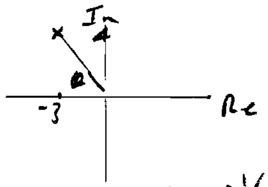
$\alpha = -3 + 4j$

i) $\alpha^2 = (-3+4j)(-3+4j)$

$= 9 - 24j - 16$

$= -7 - 24j$

ii) $|\alpha| = \sqrt{(-3)^2 + 4^2} = 5$



$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 0.927$ rad

$\arg(\alpha) = \pi - 0.927$

$= 2.214$ radians

$\alpha = 5(\cos 2.214 + j \sin 2.214)$

3)

$z^3 + z^2 - 7z - 15 = 0$

i) $3^3 + 3^2 - 7(3) - 15$

$= 27 + 9 - 21 - 15 = 0$

$\therefore z = 3$ is a root

$$\begin{array}{r} z^2 + 4z + 5 \\ z - 3 \overline{\big)\, z^3 + z^2 - 7z - 15} \\ \underline{z^3 - 3z^2 \qquad\qquad} \\ 4z^2 - 7z \qquad \\ \underline{4z^2 - 12z \qquad} \\ 5z - 15 \\ \underline{5z - 15} \end{array}$$

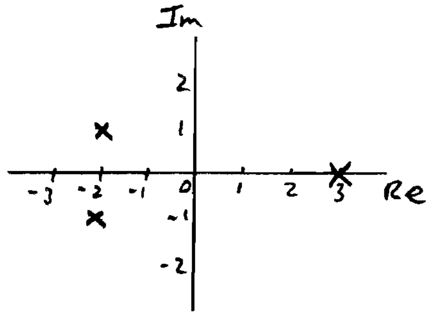
$(z-3)(z^2+4z+5) = 0$

$z = \dfrac{-4 \pm \sqrt{4^2 - 20}}{2}$

$z = \dfrac{-4 \pm 2j}{2} = -2 \pm j$

Roots $z = 3$
$z = -2 + j$
$z = -2 - j$

3ii)



4) $\sum_{r=1}^{n} \left[(r+1)(r-2) \right]$

$= \sum_{r=1}^{n} \left[r^2 - r - 2 \right]$

$= \frac{1}{6} n(n+1)(2n+1) - \frac{1}{2} n(n+1) - 2n$

$= \frac{1}{6} n \left[(n+1)(2n+1) - 3(n+1) - 12 \right]$

$= \frac{1}{6} n \left[2n^2 + 3n + 1 - 3n - 3 - 12 \right]$

4) cont)

$$= \frac{1}{6}n\left[2n^2 - 14\right]$$

$$= \frac{2}{6}n\left(n^2 - 7\right)$$

$$= \frac{1}{3}n\left(n^2 - 7\right)$$

5)

$$x^3 + px^2 + qx + r = 0$$

$$\alpha + \beta + \gamma = 3$$

$$\alpha\beta\gamma = -7$$

$$\alpha^2 + \beta^2 + \gamma^2 = 13$$

i)

$$p = -3$$

$$r = 7$$

ii

$$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2\Sigma\alpha\beta$$

$$9 = 13 + 2\Sigma\alpha\beta$$

$$2\Sigma\alpha\beta = 9 - 13 = -4$$

$$\Sigma\alpha\beta = -2$$

$$\therefore q = -2$$

6)

$$a_1 = 7, \quad a_{k+1} = 7a_k - 3$$

i)

$$a_2 = 7 \times 7 - 3 = 46$$

$$a_3 = 7 \times 46 - 3 = 319$$

ii) Prove $a_n = \dfrac{(13 \times 7^{n-1}) + 1}{2}$

When $n = 1$ $\quad a_1 = \dfrac{(13 \times 7^0) + 1}{2}$

$$\therefore a_1 = \frac{14}{2} = 7 \checkmark$$

Formula true when $n = 1$

Assume true for $n = k$

$$a_k = \frac{(13 \times 7^{k-1}) + 1}{2}$$

and prove true for $n = k+1$

$$\Rightarrow a_{k+1} = 7\left(\frac{13 \times 7^{k-1} + 1}{2}\right) - 3$$

$$a_{k+1} = \frac{13 \times 7^k + 7}{2} - 3$$

$$a_{k+1} = \frac{13 \times 7^k + 7 - 6}{2}$$

$$a_{k+1} = \frac{13 \times 7^k + 1}{2}$$

which is same formula with k replaced by $k+1$

Since formula is true for $n = 1$, by mathematical induction it is true for all positive integers n

7)

$$y = \frac{x-1}{(x-2)(x+3)(2x+3)}$$

i) When $x=0$, $y = \dfrac{-1}{(-2)(+3)(+3)}$

$= +\frac{1}{18}$

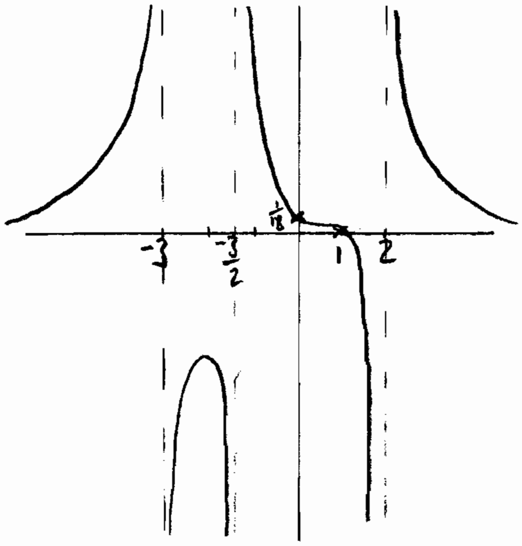
Crosses y axis at $\left(0, \frac{1}{18}\right)$

Crosses x axis at $(1, 0)$

ii) $x=2$, $x=-3$, $x=-\frac{3}{2}$

$y=0$

iii)



When $x=-1.4$ y is $\dfrac{-}{-\ +\ +}$ +ve

When $x=1.9$ y is $\dfrac{+}{-\ +\ +}$ −ve

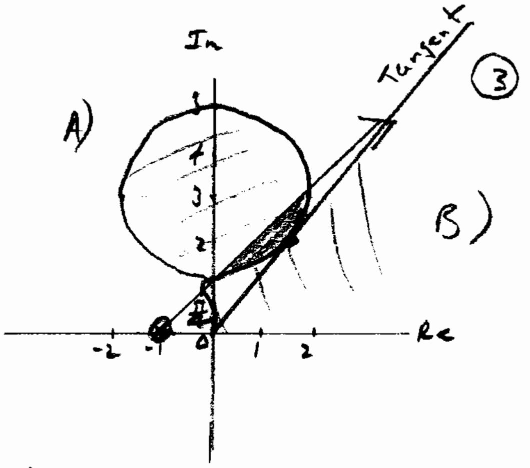
iv) Solve $y \geq 0$ From graph

$x < -3$

or $-\frac{3}{2} < x \leq 1$

or $x > 2$

8) i)



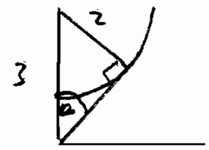
$|z-3j| \leq 2$
is a circular disc centre $3j$, radius 2

$\arg(z+1) = \arg(z-(-1)) \leq \frac{\pi}{4}$

is area between half-line and positive real axis.

ii) Shaded segment

iii) A) Point is where tangent from origin touches circle



$\sin\theta = \frac{2}{3}$

$\arg(z) = \frac{\pi}{2} - \theta$

$= \frac{\pi}{2} - \sin^{-1}\left(\frac{2}{3}\right)$

$\arg z = 0.841$ radians

9)

i) $(-3, -3)$

ii) (x, x)

iii) $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ x \end{pmatrix}$

Matrix is $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

iv) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}$

or $\begin{pmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{pmatrix}$

Rotation by $\frac{\pi}{2}$ anti-clockwise about $(0,0)$

v) MT

$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}$

Required matrix is

$\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}$

vi) $\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ x \end{pmatrix}$

All images lie on line

$y = -x$

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