RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

FRIDAY 11 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (36 marks)

1 You are given that matrix $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ and matrix $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$.

(i) Find **BA**. [2]

(ii) A plane shape of area 3 square units is transformed using matrix **A**. The image is transformed using matrix **B**. What is the area of the resulting shape? [3]

2 You are given that $\alpha = -3 + 4\mathrm{j}$.

(i) Calculate α^2. [2]

(ii) Express α in modulus-argument form. [3]

3 **(i)** Show that $z = 3$ is a root of the cubic equation $z^3 + z^2 - 7z - 15 = 0$ and find the other roots. [5]

(ii) Show the roots on an Argand diagram. [2]

4 Using the standard formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that $\sum_{r=1}^{n} [(r+1)(r-2)] = \frac{1}{3}n(n^2 - 7)$. [6]

5 The equation $x^3 + px^2 + qx + r = 0$ has roots α, β and γ, where

$$\alpha + \beta + \gamma = 3,$$
$$\alpha\beta\gamma = -7,$$
$$\alpha^2 + \beta^2 + \gamma^2 = 13.$$

(i) Write down the values of p and r. [2]

(ii) Find the value of q. [3]

6 A sequence is defined by $a_1 = 7$ and $a_{k+1} = 7a_k - 3$.

(i) Calculate the value of the third term, a_3. [2]

(ii) Prove by induction that $a_n = \frac{(13 \times 7^{n-1}) + 1}{2}$. [6]

Section B (36 marks)

7 The sketch below shows part of the graph of $y = \dfrac{x-1}{(x-2)(x+3)(2x+3)}$. One section of the graph has been omitted.

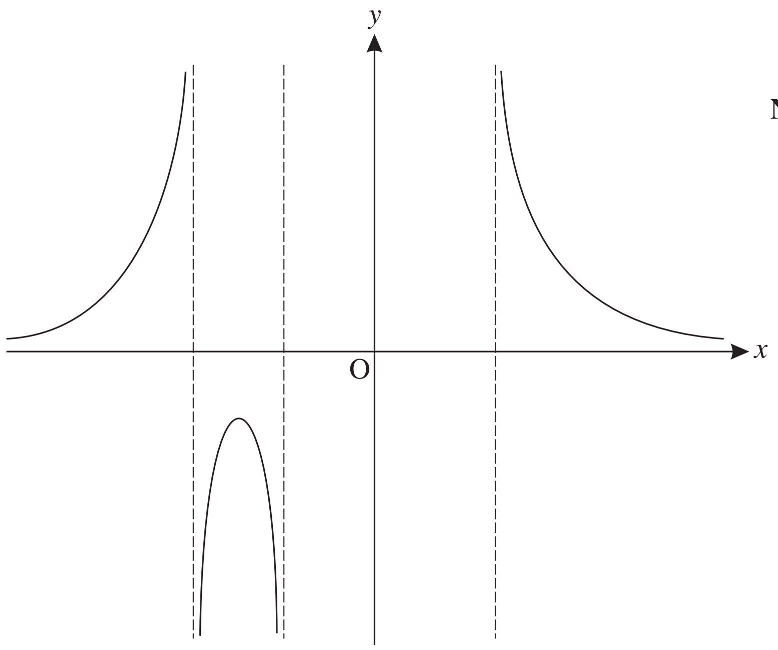


Fig. 7

(i) Find the coordinates of the points where the curve crosses the axes. [2]

(ii) Write down the equations of the three vertical asymptotes and the one horizontal asymptote. [4]

(iii) Copy the sketch and draw in the missing section. [2]

(iv) Solve the inequality $\dfrac{x-1}{(x-2)(x+3)(2x+3)} \geqslant 0$. [3]

8 **(i)** On a single Argand diagram, sketch the locus of points for which

(A) $|z - 3\mathrm{j}| = 2$, [3]

(B) $\arg(z + 1) = \frac{1}{4}\pi$. [3]

(ii) Indicate clearly on your Argand diagram the set of points for which

$$|z - 3\mathrm{j}| \leqslant 2 \quad \text{and} \quad \arg(z+1) \leqslant \tfrac{1}{4}\pi.$$ [2]

(iii) (A) By drawing an appropriate line through the origin, indicate on your Argand diagram the point for which $|z - 3\mathrm{j}| = 2$ and $\arg z$ has its minimum possible value. [2]

(B) Calculate the value of $\arg z$ at this point. [2]

9 A transformation T acts on all points in the plane. The image of a general point P is denoted by P′. P′ always lies on the line $y = x$ and has the same x-coordinate as P. This is illustrated in Fig. 9.

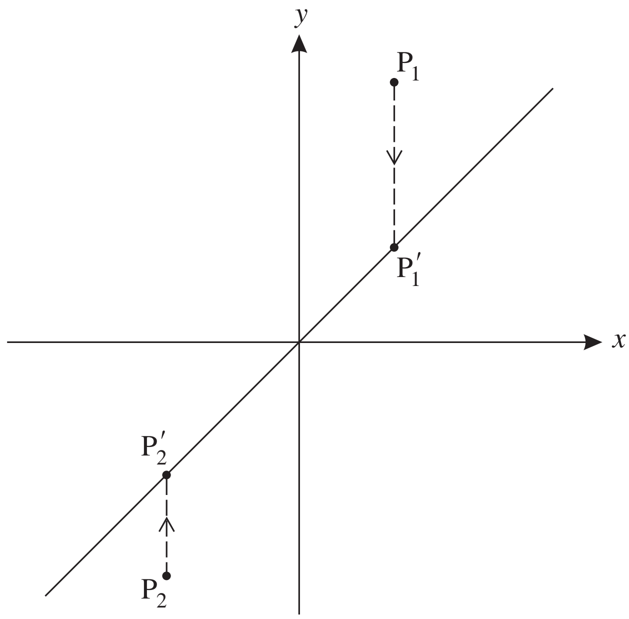


Fig. 9

(i) Write down the image of the point (−3, 7) under transformation T. [1]

(ii) Write down the image of the point (x, y) under transformation T. [2]

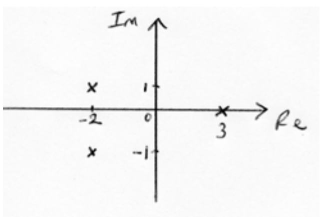
(iii) Find the 2 × 2 matrix which represents the transformation. [3]

(iv) Describe the transformation M represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [2]

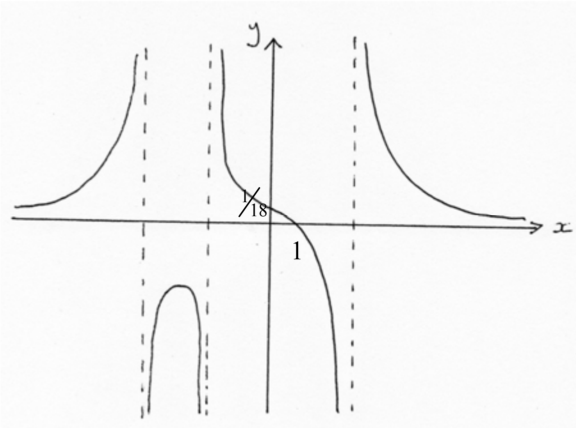
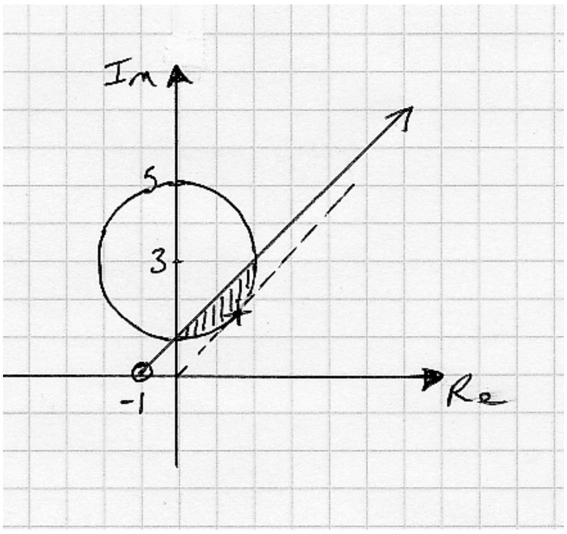
(v) Find the matrix representing the composite transformation of T followed by M. [2]

(vi) Find the image of the point (x, y) under this composite transformation. State the equation of the line on which all of these images lie. [3]

4755 (FP1) Further Concepts for Advanced Mathematics

Qu	Answer	Mark	Comment
Section A			
1(i)	$\mathbf{BA} = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ -4 & 14 \end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply c.a.o.
1(ii)	$\det \mathbf{BA} = (6\times 14) - (-4\times 0) = 84$ $3\times 84 = 252$ square units	M1 A1 A1(ft) **[3]**	Attempt to calculate any determinant c.a.o. Correct area
2(i)	$\alpha^2 = (-3+4\mathrm{j})(-3+4\mathrm{j}) = (-7-24\mathrm{j})$	M1 A1 **[2]**	Attempt to multiply with use of $\mathrm{j}^2 = -1$ c.a.o.
2(ii)	$\lvert\alpha\rvert = 5$ $\arg\alpha = \pi - \arctan\frac{4}{3}$ = 2.21 (2d.p.) (or 126.87°) $\alpha = 5(\cos 2.21 + \mathrm{j}\sin 2.21)$	B1 B1 B1(ft) **[3]**	Accept 2.2 or 127° Accept degrees and $(r,\ \theta)$ form s.c. lose 1 mark only if α^2 used throughout (ii)
3(i)	$3^3 + 3^2 - 7\times 3 - 15 = 0$ $z^3 + z^2 - 7z - 15 = (z-3)(z^2+4z+5)$ $z = \dfrac{-4\pm\sqrt{16-20}}{2} = -2\pm \mathrm{j}$ So other roots are $-2+\mathrm{j}$ and $-2-\mathrm{j}$	B1 M1 A1 M1 A1 **[5]**	Showing 3 satisfies the equation (may be implied) Valid attempt to factorise Correct quadratic factor Use of quadratic formula, or other valid method One mark for both c.a.o.
3(ii)		B2 **[2]**	Minus 1 for each error ft provided conjugate imaginary roots

4	$\sum_{r=1}^{n}\left[(r+1)(r-2)\right]=\sum_{r=1}^{n}r^2-\sum_{r=1}^{n}r-2n$ $=\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)-2n$ $=\frac{1}{6}n\left[(n+1)(2n+1)-3(n+1)-12\right]$ $=\frac{1}{6}n\left(2n^2+3n+1-3n-3-12\right)$ $=\frac{1}{6}n\left(2n^2-14\right)$ $=\frac{1}{3}n\left(n^2-7\right)$	M1 A2 M1 M1 A1 **[6]**	Attempt to split sum up Minus one each error Attempt to factorise Collecting terms All correct
5(i) **5(ii)**	$p=-3,\ r=7$ $q=\alpha\beta+\alpha\gamma+\beta\gamma$ $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $=(\alpha+\beta+\gamma)^2-2q$ $\Rightarrow 13=3^2-2q$ $\Rightarrow q=-2$	B2 **[2]** B1 M1 A1 **[3]**	One mark for each s.c. B1 if *b* and *d* used instead of *p* and *r* Attempt to find *q* using $\alpha^2+\beta^2+\gamma^2$ and $\alpha+\beta+\gamma$, but not $\alpha\beta\gamma$ c.a.o.
6(i) **6(ii)**	$a_2=7\times7-3=46$ $a_3=7\times46-3=319$ When *n* = 1, $\frac{13\times7^0+1}{2}=7$, so true for *n* = 1 Assume true for *n* = *k* $a_k=\frac{13\times7^{k-1}+1}{2}$ $\Rightarrow a_{k+1}=7\times\frac{13\times7^{k-1}+1}{2}-3$ $=\frac{13\times7^k+7}{2}-3$ $=\frac{13\times7^k+7-6}{2}$ $=\frac{13\times7^k+1}{2}$ But this is the given result with *k* + 1 replacing *k*. Therefore if it is true for *k* it is true for *k* + 1. Since it is true for *k* = 1, it is true for *k* = 1, 2, 3 and so true for all positive integers.	M1 A1 **[2]** B1 E1 M1 A1 E1 E1 **[6]**	Use of inductive definition c.a.o. Correct use of part (i) (may be implied) Assuming true for *k* Attempt to use $a_{k+1}=7a_k-3$ Correct simplification Dependent on A1 and previous E1 Dependent on B1 and previous E1
			Section A Total: 36

Section B			
7(i)	$(1,\ 0)$ and $(0,\ \frac{1}{18})$	B1 B1 [2]	
7(ii)	x = 2, $x=-3,\ x=\frac{-3}{2},\ y=0$	B4 [4]	Minus 1 for each error
7(iii)		B1 B1 [2]	Correct approaches to vertical asymptotes Through clearly marked $(1,\ 0)$ and $(0,\ \frac{1}{18})$
7(iv)	$x<-3$, $x>2$ $\frac{-3}{2}<x\le 1$	B1 B2 [3]	 B1 for $\frac{-3}{2}<x<1$, or $\frac{-3}{2}\le x\le 1$
8(i) 8(ii)		B3 B3 [6] B2(ft) [2]	Circle, B1; radius 2, B1; centre 3j, B1 Half line, B1; from -1, B1; $\frac{\pi}{4}$ to *x*-axis, B1 Correct region between their circle and half line indicated s.c. B1 for interior of circle
	Sketch should clearly show the radius and centre of the circle and the starting point and angle of the half-line.	M1 A1(ft)	Tangent from origin to circle Correct point placed by eye where tangent from origin meets circle
8(iii)	$\arg z=\frac{\pi}{2}-\arcsin\frac{2}{3}$ = 0.84 (2d.p.)	M1 A1 [4]	Attempt to use right angled triangle c.a.o. Accept 48.20° (2d.p.)

9(i)	$(-3,\ -3)$	B1 **[1]**	
9(ii)	$(x,\ x)$	B1 B1 **[2]**	
9(iii)	$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$	B3 **[3]**	Minus 1 each error to min of 0
9(iv)	Rotation through $\frac{\pi}{2}$ anticlockwise about the origin	B1 B1 **[2]**	Rotation and angle (accept 90°) Centre and sense
9(v)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}$	M1 A1 **[2]**	Attempt to multiply using their **T** in correct order c.a.o.
9(vi)	$\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ x \end{pmatrix}$ So (-*x*, *x*) Line *y* = -*x*	M1 A1(ft) A1 **[3]**	May be implied c.a.o. from correct matrix