OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4755
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Friday 20 May 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 **(i)** Write down the matrix for a rotation of 90° anticlockwise about the origin. **[1]**

(ii) Write down the matrix for a reflection in the line $y = x$. **[1]**

(iii) Find the matrix for the composite transformation of rotation of 90° anticlockwise about the origin, followed by a reflection in the line $y = x$. **[2]**

(iv) What single transformation is equivalent to this composite transformation? **[1]**

2 You are given that $z = 3 - 2\mathrm{j}$ and $w = -4 + \mathrm{j}$.

(i) Express $\dfrac{z + w}{w}$ in the form $a + b\mathrm{j}$. **[3]**

(ii) Express w in modulus-argument form. **[3]**

(iii) Show w on an Argand diagram, indicating its modulus and argument. **[2]**

3 The equation $x^3 + px^2 + qx + 3 = 0$ has roots α, β and γ, where

$$\alpha + \beta + \gamma = 4$$
$$\alpha^2 + \beta^2 + \gamma^2 = 6.$$

Find p and q. **[5]**

4 Solve the inequality $\dfrac{5x}{x^2 + 4} < x$. **[6]**

5 Given that $\dfrac{3}{(3r-1)(3r+2)} \equiv \dfrac{1}{3r-1} - \dfrac{1}{3r+2}$, find $\displaystyle\sum_{r=1}^{20} \frac{1}{(3r-1)(3r+2)}$, giving your answer as an exact fraction. **[5]**

6 Prove by induction that $1 + 8 + 27 + \ldots + n^3 = \frac{1}{4}n^2(n+1)^2$. **[7]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{(x+9)(3x-8)}{x^2-4}$.

(i) Write down the coordinates of the points where the curve crosses the axes. **[3]**

(ii) Write down the equations of the three asymptotes. **[3]**

(iii) Determine whether the curve approaches the horizontal asymptote from above or below for

(A) large positive values of x,

(B) large negative values of x. **[3]**

(iv) Sketch the curve. **[3]**

8 A polynomial P(z) has real coefficients. Two of the roots of P(z) = 0 are $2 - \mathrm{j}$ and $-1 + 2\mathrm{j}$.

(i) Explain why P(z) cannot be a cubic. **[1]**

You are given that P(z) is a quartic.

(ii) Write down the other roots of P(z) = 0 and hence find P(z) in the form $z^4 + az^3 + bz^2 + cz + d$. **[8]**

(iii) Show the roots of P(z) = 0 on an Argand diagram and give, in terms of z, the equation of the circle they lie on. **[2]**

9 The simultaneous equations

$$2x - y = 1$$
$$3x + ky = b$$

are represented by the matrix equation $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ b \end{pmatrix}$.

(i) Write down the matrix $\mathbf{M}$. **[2]**

(ii) State the value of k for which $\mathbf{M}^{-1}$ does not exist and find $\mathbf{M}^{-1}$ in terms of k when $\mathbf{M}^{-1}$ exists.

Use $\mathbf{M}^{-1}$ to solve the simultaneous equations when $k = 5$ and $b = 21$. **[7]**

(iii) What can you say about the solutions of the equations when $k = -\frac{3}{2}$? **[1]**

(iv) The two equations can be interpreted as representing two lines in the x-y plane. Describe the relationship between these two lines

(A) when $k = 5$ and $b = 21$,

(B) when $k = -\frac{3}{2}$ and $b = 1$,

(C) when $k = -\frac{3}{2}$ and $b = \frac{3}{2}$. **[3]**