

Friday 13 January 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4756 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4756
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions in Section A and **one** question from Section B.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (54 marks)

Answer all the questions

- 1 (a) A curve has polar equation $r = 1 + \cos \theta$ for $0 \leq \theta < 2\pi$.
- (i) Sketch the curve. [2]
- (ii) Find the area of the region enclosed by the curve, giving your answer in exact form. [6]
- (b) Assuming that x^4 and higher powers may be neglected, write down the Maclaurin series approximations for $\sin x$ and $\cos x$ (where x is in radians).

Hence or otherwise obtain an approximation for $\tan x$ in the form $ax + bx^3$. [6]

- (c) Find $\int_0^1 \frac{1}{\sqrt{1 - \frac{1}{4}x^2}} dx$, giving your answer in exact form. [4]

- 2 (a) The infinite series C and S are defined as follows.

$$C = 1 + a \cos \theta + a^2 \cos 2\theta + \dots,$$

$$S = a \sin \theta + a^2 \sin 2\theta + a^3 \sin 3\theta + \dots,$$

where a is a real number and $|a| < 1$.

By considering $C + jS$, show that $C = \frac{1 - a \cos \theta}{1 + a^2 - 2a \cos \theta}$ and find a corresponding expression for S . [8]

- (b) Express the complex number $z = -1 + j\sqrt{3}$ in the form $r e^{j\theta}$.

Find the 4th roots of z in the form $r e^{j\theta}$.

Show z and its 4th roots in an Argand diagram.

Find the product of the 4th roots and mark this as a point on your Argand diagram. [10]

- 3 (i) Show that the characteristic equation of the matrix

$$\mathbf{M} = \begin{pmatrix} 3 & -1 & 2 \\ -4 & 3 & 2 \\ 2 & 1 & -1 \end{pmatrix}$$

is $\lambda^3 - 5\lambda^2 - 7\lambda + 35 = 0$. [4]

- (ii) Show that $\lambda = 5$ is an eigenvalue of \mathbf{M} , and find its other eigenvalues. [4]

- (iii) Find an eigenvector, \mathbf{v} , of unit length corresponding to $\lambda = 5$.

State the magnitudes and directions of the vectors $\mathbf{M}^2\mathbf{v}$ and $\mathbf{M}^{-1}\mathbf{v}$. [6]

- (iv) Use the Cayley-Hamilton theorem to find the constants a , b , c such that

$$\mathbf{M}^4 = a\mathbf{M}^2 + b\mathbf{M} + c\mathbf{I}. [4]$$

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 (i) Define $\tanh t$ in terms of exponential functions. Sketch the graph of $\tanh t$. [3]

(ii) Show that $\operatorname{artanh} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$. State the set of values of x for which this equation is valid. [5]

(iii) Differentiate the equation $\tanh y = x$ with respect to x and hence show that the derivative of $\operatorname{artanh} x$ is $\frac{1}{1-x^2}$.

Show that this result may also be obtained by differentiating the equation in part (ii). [5]

(iv) By considering $\operatorname{artanh} x$ as $1 \times \operatorname{artanh} x$ and using integration by parts, show that

$$\int_0^{\frac{1}{2}} \operatorname{artanh} x \, dx = \frac{1}{4} \ln \frac{27}{16}. \quad [5]$$

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 The points A(-1, 0), B(1, 0) and P(x, y) are such that the product of the distances PA and PB is 1. You are given that the cartesian equation of the locus of P is

$$((x+1)^2 + y^2)((x-1)^2 + y^2) = 1.$$

(i) Show that this equation may be written in polar form as

$$r^4 + 2r^2 = 4r^2 \cos^2 \theta.$$

Show that the polar equation simplifies to

$$r^2 = 2 \cos 2\theta. \quad [4]$$

(ii) Give a sketch of the curve, stating the values of θ for which the curve is defined. [4]

(iii) The equation in part (i) is now to be generalised to

$$r^2 = 2 \cos 2\theta + k,$$

where k is a constant.

(A) Give sketches of the curve in the cases $k = 1$, $k = 2$. Describe how these two curves differ at the pole.

(B) Give a sketch of the curve in the case $k = 4$. What happens to the shape of the curve as k tends to infinity? [7]

(iv) Sketch the curve for the case $k = -1$.

What happens to the curve as $k \rightarrow -2$? [3]

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.



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