FP2 JANUARY 2010 Q2 COMPLEX NUMBERS

2 a) By de Moivre's theorem

$$(\cos\theta + j\sin\theta)^5 = \cos 5\theta + j\sin 5\theta$$

Let $c = \cos\theta$, $s = \sin\theta$

1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$(c+js)^5 =$

$$c^5 + 5c^4js + 10c^3j^2s^2 + 10c^2j^3s^3 + 5cj^4s^4 + j^5s^5$$

$$= c^5 + 5jc^4s - 10c^3s^2 - 10jc^2s^3 + 5cs^4 + js^5$$

Equating real and imaginary parts

$$\cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$$

$$= c^5 - 10c^3(1-c^2) + 5c(1-c^2)^2$$

$$= c^5 - 10c^3 + 10c^5 + 5c(1 - 2c^2 + c^4)$$

$$= c^5 - 10c^3 + 10c^5 + 5c - 10c^3 + 5c^5$$

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

2b) $C = \cos\theta + \cos\left(\theta + \frac{2\pi}{n}\right) + \cos\left(\theta + \frac{4\pi}{n}\right) + \ldots + \cos\left(\theta + \frac{(2n-2)\pi}{n}\right)$

$S = \sin\theta + \sin\left(\theta + \frac{2\pi}{n}\right) + \sin\left(\theta + \frac{4\pi}{n}\right) + \ldots + \sin\left(\theta + \frac{(2n-2)\pi}{n}\right)$

$C + jS = e^{j\theta} + e^{j(\theta + \frac{2\pi}{n})} + e^{j(\theta + \frac{4\pi}{n})} + \ldots + e^{j(\theta + \frac{(2n-2)\pi}{n})}$

This is a GP with n terms

1st term $a = e^{j\theta}$, $r = e^{j\frac{2\pi}{n}}$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{e^{j\theta}(1 - e^{j2\pi})}{1 - e^{j\frac{2\pi}{n}}}$$

$$S_n = \frac{e^{j\theta}(1-1)}{1 - e^{j\frac{2\pi}{n}}} = 0$$

FP2 JANUARY 2010 Q2 COMPLEX NUMBERS

2b) cont

$$\therefore \quad C + jS = 0$$

Equating real and imaginary parts gives

$$C = 0 \quad , \quad S = 0$$

2c)

$$e^t = 1 + t + \frac{t^2}{2!} + \dots$$

$$e^t \approx 1 + t + \frac{t^2}{2} \qquad \text{for } t \text{ close to } 0$$

$$\frac{t}{e^t - 1} \approx \frac{t}{1 + t + \frac{t^2}{2} - 1}$$

$$\approx \frac{t}{t + \frac{t^2}{2}}$$

$$\approx \frac{1}{1 + \frac{t}{2}}$$

$$\approx \left(1 + \frac{t}{2}\right)^{-1}$$

$$\approx 1 - \frac{t}{2}$$

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