FP2 JANUARY 2012 Q2 COMPLEX NUMBERS

2 a)

$$C = 1 + a\cos\theta + a^2\cos 2\theta + \dots$$

$$S = a\sin\theta + a^2\sin 2\theta + a^3\sin 3\theta + \dots$$

$$C + jS = 1 + a(\cos\theta + j\sin\theta) + a^2(\cos 2\theta + j\sin 2\theta) + \dots$$

$$C + jS = 1 + ae^{j\theta} + a^2e^{j2\theta} + \dots$$

Since $|a| < 1$ this is a GP with a sum to infinity

first term $a = 1$, $r = ae^{j\theta}$

for GP $S_\infty = \dfrac{a}{1-r} = \dfrac{1}{1-ae^{j\theta}}$

$$C + jS = \frac{1}{1-ae^{j\theta}} \times \frac{1-ae^{-j\theta}}{1-ae^{-j\theta}}$$

$$= \frac{1-ae^{-j\theta}}{1-ae^{j\theta}-ae^{-j\theta}+a^2}$$

$$= \frac{1-ae^{-j\theta}}{1-a(e^{j\theta}+e^{-j\theta})+a^2}$$

$$= \frac{1-a(\cos\theta - j\sin\theta)}{1-2a\cos\theta+a^2}$$

$$= \frac{1-a\cos\theta+aj\sin\theta}{1-2a\cos\theta+a^2}$$

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2a cont)

$$C + jS = \frac{1 - a\cos\theta + aj\sin\theta}{1 - 2a\cos\theta + a^2}$$

Equating real and imaginary parts

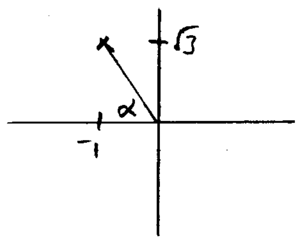
$$C = \frac{1 - a\cos\theta}{1 - 2a\cos\theta + a^2}$$

$$S = \frac{a\sin\theta}{1 - 2a\cos\theta + a^2}$$

2b)

$$z = -1 + j\sqrt{3}$$

$$|z| = \sqrt{(-1)^2 + \sqrt{3}^2} = 2$$



$$\alpha = \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{3}$$

$$\arg(z) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$z = 2e^{j\frac{2\pi}{3}}$$

4th roots $\quad 2^{\frac{1}{4}} e^{j\left(\frac{2\pi}{12} + \frac{2n\pi}{4}\right)}$ for $n = 0, 1, 2, 3$

$$2^{\frac{1}{4}} e^{j\frac{\pi}{6}},\ 2^{\frac{1}{4}} e^{j\left(\frac{\pi}{6} + \frac{\pi}{2}\right)},\ 2^{\frac{1}{4}} e^{j\left(\frac{\pi}{6} + \pi\right)},\ 2^{\frac{1}{4}} e^{j\left(\frac{\pi}{6} + \frac{3\pi}{2}\right)}$$

$$= 2^{\frac{1}{4}} e^{j\frac{\pi}{6}},\ 2^{\frac{1}{4}} e^{j\frac{2\pi}{3}},\ 2^{\frac{1}{4}} e^{-j\frac{5\pi}{6}},\ 2^{\frac{1}{4}} e^{-j\frac{\pi}{3}}$$

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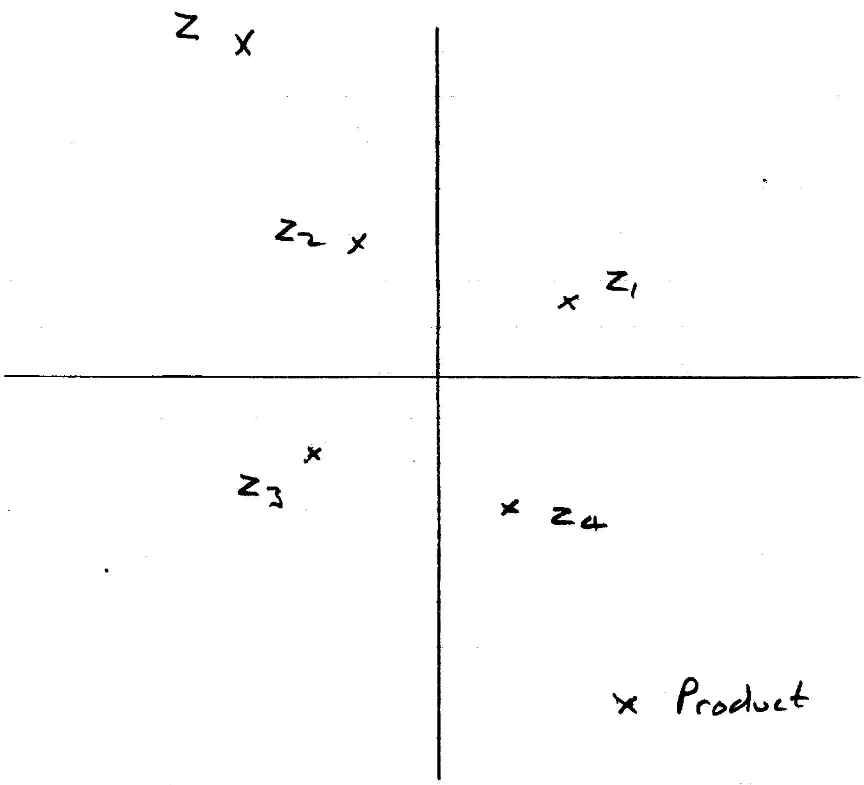
2b cont)



$|z_1| \approx 1.18$
ditto for $z_2\ z_3\ z_4$

$$\text{Product} = \left(2^{\frac{1}{4}}\right)^4 e^{j\left(\frac{\pi}{6} + \frac{2\pi}{3} - \frac{5\pi}{6} - \frac{\pi}{3}\right)}$$

$$= 2\, e^{-j\frac{\pi}{3}}$$