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2a)

i) By de Moivre's theorem

$$(\cos\theta + j\sin\theta)^5 = \cos 5\theta + j\sin 5\theta$$

Let $c = \cos\theta$, $s = \sin\theta$

$$(c+js)^5 = c^5 + 5c^4 js + 10c^3 j^2 s^2 + 10c^2 j^3 s^3 + 5cj^4 s^4 + j^5 s^5$$

$$= c^5 + 5c^4 sj - 10c^3 s^2 - 10c^2 s^3 j + 5cs^4 + s^5 j$$

Equating real and imaginary parts

$$\cos 5\theta = c^5 - 10c^3 s^2 + 5cs^4$$

$$= c^5 - 10c^3(1-c^2) + 5c(1-c^2)^2$$

$$= c^5 - 10c^3 + 10c^5 + 5c(1 - 2c^2 + c^4)$$

$$= c^5 - 10c^3 + 10c^5 + 5c - 10c^3 + 5c^5$$

$$= 16c^5 - 20c^3 + 5c$$

$$\therefore \quad \cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

ii) If $\cos 5\theta = 0$ but $\cos\theta \neq 0$

$$0 = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

$$0 = \cos\theta\left(16\cos^4\theta - 20\cos^2\theta + 5\right)$$

$$\cos^2\theta = \frac{20 \pm \sqrt{400 - 320}}{32}$$

$$\cos^2\theta = \frac{20 \pm \sqrt{80}}{32} = \frac{20 \pm 4\sqrt{5}}{32} = \frac{5 \pm \sqrt{5}}{8}$$

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2 a ii cont) When $\theta = 18^\circ$, $\cos 5\theta = \cos 90^\circ = 0$ and $\cos 18^\circ \neq 0$

From previous argument

$$\cos^2 18^\circ = \frac{5 \pm \sqrt{5}}{8}$$

$$\Rightarrow \cos 18^\circ = \sqrt{\frac{5 \pm \sqrt{5}}{8}}$$

Since $\theta = 18^\circ$ is the smallest positive angle for which $\cos 5\theta = 0$

$\cos 18^\circ$ will take the larger of the two possible values

$$\Rightarrow \cos 18^\circ = \sqrt{\frac{5 + \sqrt{5}}{8}}$$

$$\sin^2 18 = 1 - \cos^2 18$$

$$= 1 - \left(\frac{5 + \sqrt{5}}{8}\right)$$

$$= \frac{8 - 5 - \sqrt{5}}{8}$$

$$= \frac{3 - \sqrt{5}}{8}$$

$$\therefore \sin 18^\circ = \sqrt{\frac{3 - \sqrt{5}}{8}}$$

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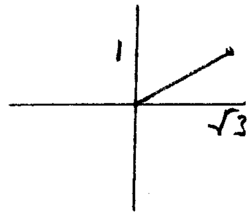
2b) i) $\left|4(\sqrt{3}+j)\right| = 4\sqrt{\sqrt{3}^2+1^2} = 8$

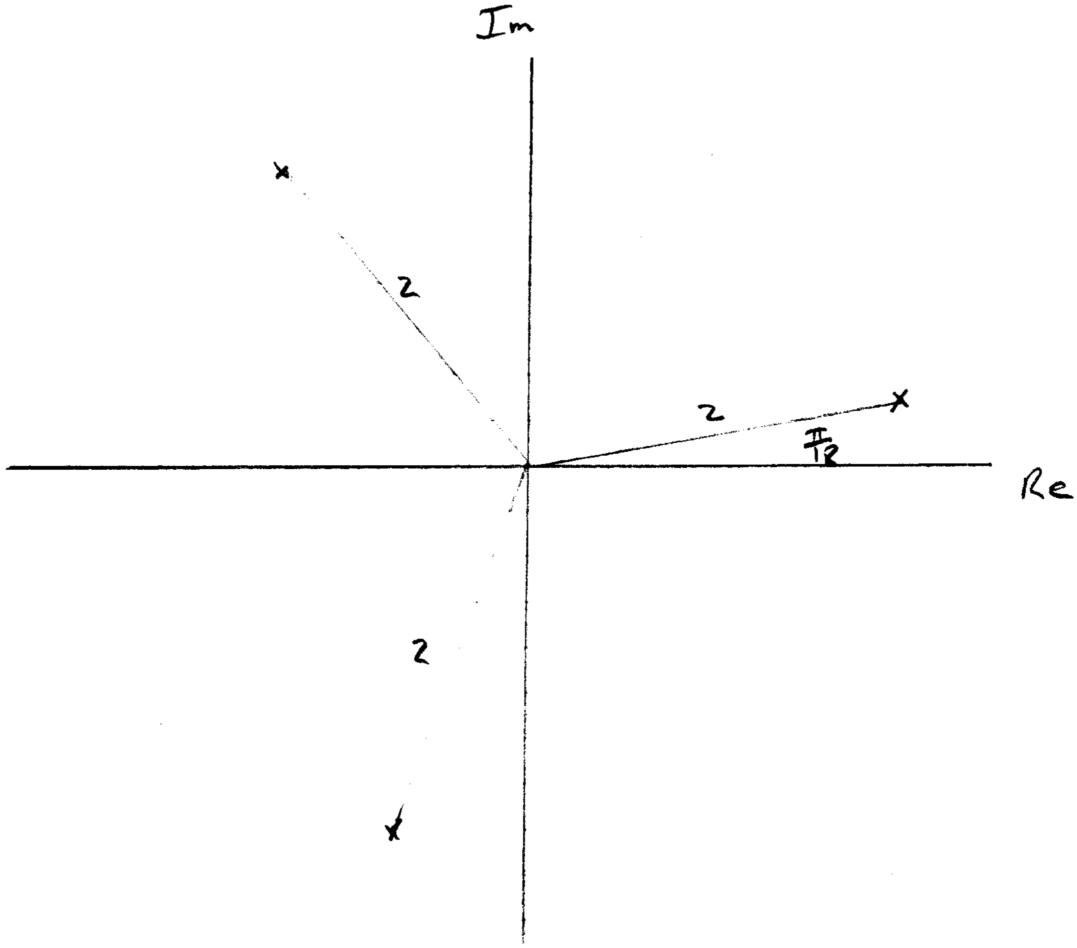
$\arg(4(\sqrt{3}+j)) = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$



$\therefore\ 4(\sqrt{3}+j) = 8e^{j\frac{\pi}{6}}$

Cube roots are $8^{\frac{1}{3}}e^{j\frac{\frac{\pi}{6}+2n\pi}{3}}$ for $n = 0, 1, 2$

$= \quad 2e^{j\frac{\pi}{18}}, \quad 2e^{j\frac{13\pi}{18}}, \quad 2e^{j\frac{25\pi}{18}}$



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2bii) $\arg(w) = \frac{\pi}{18} + \frac{\frac{2\pi}{3}}{2}$

$= \frac{\pi}{18} + \frac{\pi}{3}$

$= \frac{7\pi}{18}$

$\arg(w) = \frac{7\pi}{18}$

For real w^n, $\arg(w^n)$ an integer multiple of π

$\Rightarrow \quad n = 18$