FP2 JANUARY 2010 Q4 HYPERBOLIC FUNCTIONS

4 i) Prove $\cosh 2x = 1 + 2\sinh^2 x$

$$1 + 2\sinh^2 x = 1 + 2\left(\tfrac{1}{2}(e^x - e^{-x})\right)^2$$
$$= 1 + \tfrac{1}{2}\left(e^{2x} - 2 + e^{-2x}\right)$$
$$= 1 - 1 + \tfrac{1}{2}\left(e^{2x} + e^{-2x}\right)$$
$$= \cosh 2x$$

$$\cosh 2x = 1 + 2\sinh^2 x = 1 + 2(\sinh x)^2$$

d w.r.t. x

$$2\sinh 2x = 4\sinh x \cosh x$$
$$\Rightarrow \quad \sinh 2x = 2\sinh x \cosh x$$

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ii)

$$2\cosh 2x + 3\sinh x = 3$$
$$2 + 4\sinh^2 x + 3\sinh x - 3 = 0$$
$$4\sinh^2 x + 3\sinh x - 1 = 0$$
$$(4\sinh x - 1)(\sinh x + 1) = 0$$
$$\Rightarrow \quad \sinh x = \frac{1}{4} \quad \text{or} \quad \sinh x = -1$$
$$\Rightarrow \quad x = \operatorname{arsinh}\left(\tfrac{1}{4}\right) \quad \text{or} \quad x = \operatorname{arsinh}(-1)$$
$$x = \ln\left(\tfrac{1}{4} + \sqrt{\tfrac{1}{16} + 1}\right) \quad \text{or} \quad x = \ln\left(-1 + \sqrt{2}\right)$$
$$x = \ln\left(\frac{1 + \sqrt{17}}{4}\right) \quad \text{or} \quad x = \ln\left(\sqrt{2} - 1\right)$$

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4iii) Given $\cosh t = \frac{5}{4}$

$\Rightarrow \quad \frac{1}{2}\left(e^{t} + e^{-t}\right) = \frac{5}{4}$

$\Rightarrow \quad 2e^{t} + 2e^{-t} = 5 \qquad \times e^{t}$

$\Rightarrow \quad 2e^{2t} + 2 = 5e^{t}$

$\Rightarrow \quad 2e^{2t} - 5e^{t} + 2 = 0$

$\Rightarrow \quad e^{t} = \dfrac{5 \pm \sqrt{25-16}}{4}$

$\Rightarrow \quad e^{t} = \dfrac{5 \pm 3}{4}$

$\Rightarrow \quad e^{t} = 2 \quad \text{or} \quad e^{t} = \frac{1}{2}$

$\Rightarrow \quad t = \ln 2 \quad \text{or} \quad t = \ln\left(\frac{1}{2}\right)$

$\Rightarrow \quad t = \ln 2 \quad \text{or} \quad t = -\ln 2$

$$\int_4^5 \frac{1}{\sqrt{x^2-16}}\,dx = \left[\operatorname{arcosh}\left(\frac{x}{4}\right)\right]_4^5$$

$$= \operatorname{arcosh}\left(\frac{5}{4}\right) - \operatorname{arcosh}(1)$$

$$= \ln 2 - 0$$

$$= \ln 2$$

(Choosing $\ln 2$ rather than $-\ln 2$ since area under curve is above x-axis)