4 i)

$$\sinh t + 7\cosh t = 8$$

$$\tfrac{1}{2}(e^t - e^{-t}) + \tfrac{7}{2}(e^t + e^{-t}) = 8$$

$$e^t - e^{-t} + 7e^t + 7e^{-t} = 16$$

$$8e^t + 6e^{-t} = 16$$

$$4e^t + 3e^{-t} = 8$$

$$4e^{2t} + 3 = 8e^t$$

$$4e^{2t} - 8e^t + 3 = 0$$

$$(2e^t - 1)(2e^t - 3) = 0$$

$$\Rightarrow \quad 2e^t = 1 \quad \text{or} \quad 2e^t = 3$$

$$e^t = \tfrac{1}{2} \qquad e^t = \tfrac{3}{2}$$

$$\Rightarrow \quad t = \ln\left(\tfrac{1}{2}\right) \quad \text{or} \quad t = \ln\left(\tfrac{3}{2}\right)$$

4 ii)

$$y = \cosh 2x + 7\sinh 2x$$

$$\frac{dy}{dx} = 2\sinh 2x + 14\cosh 2x$$

If gradient $= 16$ then $2\sinh 2x + 14\cosh 2x = 16$

$$\sinh 2x + 7\cosh 2x = 8$$

$$\Rightarrow \quad 2x = \ln\left(\tfrac{1}{2}\right) \quad \text{or} \quad 2x = \ln\left(\tfrac{3}{2}\right)$$

$$x = \tfrac{1}{2}\ln\left(\tfrac{1}{2}\right) \quad \text{or} \quad x = \tfrac{1}{2}\ln\left(\tfrac{3}{2}\right)$$

When $x = \frac{1}{2}\ln\left(\frac{1}{2}\right)$

$$y = \cosh\left(\ln\tfrac{1}{2}\right) + 7\sinh\left(\ln\tfrac{1}{2}\right)$$

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4ii) cont

$$y = \frac{1}{2}\left(e^{\ln\frac{1}{2}} + e^{-\ln\frac{1}{2}}\right) + \frac{7}{2}\left(e^{\ln\frac{1}{2}} - e^{-\ln\frac{1}{2}}\right)$$

$$= \frac{1}{2}\left(\frac{1}{2} + \frac{1}{\frac{1}{2}}\right) + \frac{7}{2}\left(\frac{1}{2} - \frac{1}{\frac{1}{2}}\right)$$

$$= \frac{1}{2}\left(\frac{5}{2}\right) + \frac{7}{2}\left(-\frac{3}{2}\right)$$

$$= \frac{5}{4} - \frac{21}{4}$$

$$= -4$$

Point $\left(\ln\left(\frac{1}{2}\right), -4\right)$

When $x = \ln\left(\frac{3}{2}\right)$

$$y = \frac{1}{2}\left(e^{\ln\frac{3}{2}} + e^{-\ln\frac{3}{2}}\right) + \frac{7}{2}\left(e^{\ln 3/2} - e^{-\ln 3/2}\right)$$

$$= \frac{1}{2}\left(\frac{3}{2} + \frac{2}{3}\right) + \frac{7}{2}\left(\frac{3}{2} - \frac{2}{3}\right)$$

$$= \frac{1}{2}\left(\frac{13}{6}\right) + \frac{7}{2}\left(\frac{5}{6}\right)$$

$$= \frac{13}{12} + \frac{35}{12}$$

$$= \frac{48}{12} = 4$$

Point $\left(\ln\left(\frac{3}{2}\right), 4\right)$

4ii cont) If $\frac{dy}{dx} = 0$ then $2\sinh 2x + 14\cosh 2x = 0$

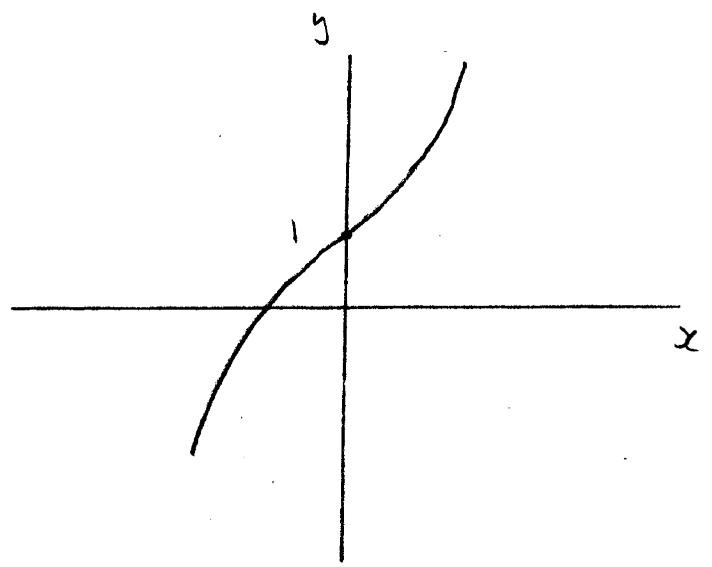
$$\sinh 2x + 7\cosh 2x = 0$$

$$\sinh 2x = -7\cosh 2x$$

$$\frac{\sinh 2x}{\cosh 2x} = \tanh 2x = -7$$

No solution since $-1 < \tanh 2x < 1$ for all x

$\therefore$ gradient is never 0



4 iii) $\int_0^a y\,dx = \frac{1}{2}$

$$\int_0^a (\cosh 2x + 7\sinh 2x)\,dx = \frac{1}{2}$$

$$\left[\frac{1}{2}\sinh 2x + \frac{7}{2}\cosh 2x\right]_0^a = \frac{1}{2}$$

$$\frac{1}{2}\sinh 2a + \frac{7}{2}\cosh 2a - \left(\frac{1}{2}\sinh 0 + \frac{7}{2}\cosh 0\right) = \frac{1}{2}$$

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4iii cont)

$$\frac{1}{2}\sinh 2a + \frac{7}{2}\cosh 2a - \left(0 + \frac{7}{2}\right) = \frac{1}{2}$$

$$\frac{1}{2}\sinh 2a + \frac{7}{2}\cosh 2a = 4$$

$$\sinh 2a + 7\cosh 2a = 8$$

From part (i) $2a = \ln\left(\frac{1}{2}\right)$ or $2a = \ln\left(\frac{3}{2}\right)$

$\Rightarrow$ $a = \frac{1}{2}\ln\left(\frac{1}{2}\right)$ or $a = \frac{1}{2}\ln\left(\frac{3}{2}\right)$

Reject $a = \frac{1}{2}\ln\left(\frac{1}{2}\right)$ since < 0 and therefore

$$a = \frac{1}{2}\ln\left(\frac{3}{2}\right)$$