FP2 JUNE 2011 Q4 HYPERBOLIC FUNCTIONS

4 i) $\cosh y = x$

$\Rightarrow \frac{1}{2}(e^y + e^{-y}) = x$

$\Rightarrow e^y + e^{-y} = 2x$

$\Rightarrow e^{2y} + 1 = 2xe^y$

$\Rightarrow e^{2y} - 2xe^y + 1 = 0$

$\Rightarrow e^y = \dfrac{2x \pm \sqrt{4x^2 - 4}}{2}$

$\Rightarrow e^y = \dfrac{2x \pm 2\sqrt{x^2-1}}{2}$

$\Rightarrow e^y = x \pm \sqrt{x^2-1}$

$\Rightarrow y = \ln\left(x \pm \sqrt{x^2-1}\right)$

However, $(x + \sqrt{x^2-1})(x - \sqrt{x^2-1})$

$= x^2 - (x^2 - 1)$ diff of 2 squares

$= 1$

$\therefore x - \sqrt{x^2-1} = \dfrac{1}{x + \sqrt{x^2-1}}$

$\Rightarrow \ln\left(x - \sqrt{x^2-1}\right) = \ln\left(\dfrac{1}{x + \sqrt{x^2-1}}\right)$

$= -\ln\left(x + \sqrt{x^2-1}\right)$

Thus we can write

$y = \pm\ln\left(x + \sqrt{x^2-1}\right)$

4i) cont) Since $\cosh y = x$

$$y = \operatorname{arcosh} x$$

$$\therefore \quad \operatorname{arcosh} x = \ln\left(x + \sqrt{x^2 - 1}\right)$$

taking the positive value to be the principal value

4ii)

$$\int_{\frac{4}{5}}^{1} \frac{1}{\sqrt{25x^2 - 16}} = \int_{\frac{4}{5}}^{1} \frac{1}{\sqrt{25\left(x^2 - \frac{16}{25}\right)}}\, dx$$

$$= \frac{1}{5}\int_{\frac{4}{5}}^{1} \frac{1}{\sqrt{x^2 - \left(\frac{4}{5}\right)^2}}\, dx$$

$$= \frac{1}{5}\left[\ln\left(x + \sqrt{x^2 - \left(\tfrac{4}{5}\right)^2}\right.\right]_{\frac{4}{5}}^{1}$$

$$= \frac{1}{5}\left[\ln\left(1 + \sqrt{1 - \left(\tfrac{4}{5}\right)^2}\right) - \ln\left(\frac{4}{5} - \sqrt{\left(\tfrac{4}{5}\right)^2 - \left(\tfrac{4}{5}\right)^2}\right)\right]$$

$$= \frac{1}{5}\left[\ln\left(1 + \frac{3}{5}\right) - \ln\left(\frac{4}{5}\right)\right]$$

$$= \frac{1}{5}\ln\left(\frac{8/5}{4/5}\right) = \frac{1}{5}\ln 2$$

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4 iii)

$$5\cosh x - \cosh 2x = 3$$

$$5\cosh x - (2\cosh^2 x - 1) = 3$$

$$5\cosh x - 2\cosh^2 x + 1 = 3$$

$$0 = 2\cosh^2 x - 5\cosh x + 2$$

$$0 = (2\cosh x - 1)(\cosh x - 2)$$

$\Rightarrow \quad \cosh x = \frac{1}{2} \quad$ or $\quad \cosh x = 2$

$\Rightarrow \quad x = \operatorname{arcosh}\left(\frac{1}{2}\right) \quad$ or $\quad x = \operatorname{arcosh}(2)$

$\Rightarrow \quad$ no solution $\quad$ or $\quad x = \pm \ln\left(2 + \sqrt{3}\right)$