FP2 JANUARY 2013 Q4 HYPERBOLIC FUNCTIONS

4 i) $y = 3\sinh x - 2\cosh x$

$\Rightarrow \frac{dy}{dx} = 3\cosh x - 2\sinh x$

At t.p. $\frac{dy}{dx} = 0 \Rightarrow 3\cosh x - 2\sinh x = 0$

$3\cosh x = 2\sinh x$

$\frac{3}{2} = \frac{\sinh x}{\cosh x} = \tanh x$

$\Rightarrow x = \operatorname{artanh} \frac{3}{2}$

no solution since $-1 < \tanh x < 1$ for all x

$\therefore$ no turning points

$y = 3\sinh x - 2\cosh x$

$y = \frac{3}{2}\left(e^x - e^{-x}\right) - \frac{2}{2}\left(e^x + e^{-x}\right)$

When $x = \frac{1}{2}\ln 5 = \ln\sqrt{5}$

$y = \frac{3}{2}\left(e^{\ln\sqrt{5}} - e^{-\ln\sqrt{5}}\right) - \left(e^{\ln\sqrt{5}} + e^{-\ln\sqrt{5}}\right)$

$y = \frac{3}{2}\left(\sqrt{5} - \frac{1}{\sqrt{5}}\right) - \left(\sqrt{5} + \frac{1}{\sqrt{5}}\right)$

$y = \frac{\sqrt{5}}{2} - \frac{5}{2\sqrt{5}} = \frac{\sqrt{5}}{2} - \frac{\sqrt{5}}{2} = 0$

$\therefore$ crosses x-axis at $x = \frac{1}{2}\ln 5$

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4i) cont)

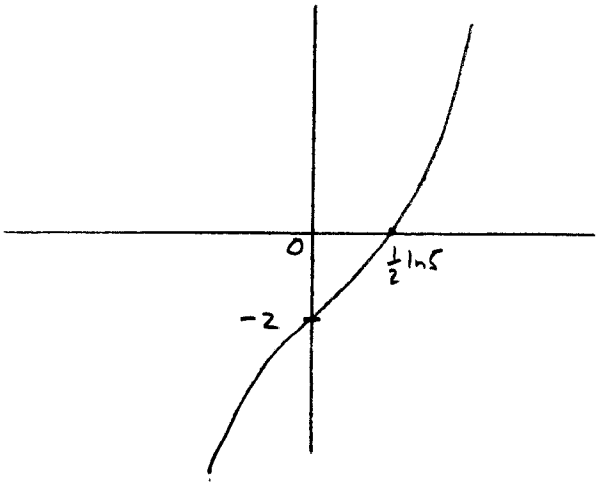
$$\frac{d^2y}{dx^2} = 3\sinh x - 2\cosh x = y$$

so $y = 0 \Rightarrow \frac{d^2y}{dx^2} = 0$

$y = 0$ at $x = \frac{1}{2}\ln 5$ so $\frac{d^2y}{dx^2} = 0$ at $x = \frac{1}{2}\ln 5$

so gradient of curve has stationary value when $x = \frac{1}{2}\ln 5$

4ii)



4iii) $(3\sinh x - 2\cosh x)^2 = 9\sinh^2 x - 12\sinh x\cosh x + 4\cosh^2 x$

$= 5\sinh^2 x + 4(\sinh^2 x + \cosh^2 x) - 12\sinh x\cosh x$

$= 5\sinh^2 x + 4\cosh 2x - 6\sinh 2x$

$= \frac{5}{2}(\cosh 2x - 1) + 4\cosh 2x - 6\sinh 2x$

$= \frac{13}{2}\cosh 2x - 6\sinh 2x - \frac{5}{2}$

Now
$\cosh 2x = 1 + 2\sinh^2 x$
$\cosh 2x - 1 = 2\sinh^2 x$
$\frac{\cosh 2x - 1}{2} = \sinh^2 x$

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4 iii cont) Volume $= \pi \int_0^{\frac{1}{2}\ln 5} y^2 \, dx$

$$= \pi \int_0^{\frac{1}{2}\ln 5} \left(\frac{13}{2}\cosh 2x - 6\sinh 2x - \frac{5}{2} \right)$$

$$= \pi \left[\frac{13}{4}\sinh 2x - 3\cosh 2x - \frac{5}{2}x \right]_0^{\frac{1}{2}\ln 5}$$

$$= \pi \left[\frac{13}{4}\sinh(\ln 5) - 3\cosh(\ln 5) - \frac{5}{4}\ln 5 - \left(\frac{13}{4}\sinh 0 - 3\cosh 0 - 0 \right) \right]$$

$$= \pi \left[\frac{13}{4}\left(\frac{1}{2}\left(5 - \frac{1}{5}\right)\right) - 3\left(\frac{1}{2}\left(5 + \frac{1}{5}\right)\right) - \frac{5}{4}\ln 5 + 3 \right]$$

$$= \pi \left[\frac{13}{8} \times \frac{24}{5} - \frac{3}{2} \times \frac{26}{5} + 3 - \frac{5}{4}\ln 5 \right]$$

$$= \pi \left(3 - \frac{5}{4}\ln 5 \right)$$