FP2 JUNE 2014 Q4 HYPERBOLIC FUNCTIONS

4 i)

$$\sinh y = x$$

$$\tfrac{1}{2}(e^{y} - e^{-y}) = x$$

$$e^{y} - e^{-y} = 2x$$

$$e^{2y} - 1 = 2xe^{y}$$

$\times e^{y}$

$$e^{2y} - 2xe^{y} - 1 = 0$$

$$e^{y} = \frac{2x \pm \sqrt{4x^2+4}}{2}$$

$$e^{y} = x \pm \sqrt{1+x^2}$$

$$y = \ln(x \pm \sqrt{1+x^2})$$

$$y = \ln(x + \sqrt{1+x^2})$$

since $x - \sqrt{1+x^2} < 0$ so log not defined

$$y = \ln(x + \sqrt{1+x^2})$$

$$e^{y} = x + (1+x^2)^{\frac{1}{2}}$$

$$e^{y}\frac{dy}{dx} = 1 + \tfrac{1}{2}(1+x^2)^{-\frac{1}{2}} \times 2x$$

$$e^{y}\frac{dy}{dx} = 1 + \frac{x}{\sqrt{1+x^2}}$$

$$\frac{dy}{dx} = \frac{1 + \frac{x}{\sqrt{1+x^2}}}{x + \sqrt{1+x^2}}$$

4: i) cont

$$\frac{dy}{dx} = \frac{1 + \frac{x}{\sqrt{1+x^2}}}{x + \sqrt{1+x^2}} \times \frac{\sqrt{1+x^2} - x}{\sqrt{1+x^2} - x}$$

$$\frac{dy}{dx} = \frac{\sqrt{1+x^2} \cancel{+x} \cancel{-x} - \frac{x^2}{\sqrt{1+x^2}}}{(1+x^2) - x^2}$$

$$\frac{dy}{dx} = \frac{\frac{1+x^2-x^2}{\sqrt{1+x^2}}}{1}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$$

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ii) $$\int \frac{1}{\sqrt{25+4x^2}}\,dx = \int \frac{1}{\sqrt{4\left(\frac{25}{4}+x^2\right)}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{5}{2}\right)^2+x^2}}\,dx$$

$$= \frac{1}{2}\ln\left(x + \sqrt{x^2+\left(5/2\right)^2}\right) + c$$

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4 iii) $\int \sqrt{25+4x^2}\,dx$

Let $2x = 5\sinh u$

$2\frac{dx}{du} = 5\cosh u$

$dx = \frac{5}{2}\cosh u$

$$= \int \sqrt{25 + 25\sinh^2 u} \times \frac{5}{2}\cosh u \, du$$

$$= \frac{25}{2}\int \sqrt{1+\sinh^2 u} \times \cosh u \, du$$

$$= \frac{25}{2}\int \cosh^2 u \, du$$

$$= \frac{25}{2}\int \frac{\cosh 2u + 1}{2} \, du$$

$$= \frac{25}{4}\left[\frac{1}{2}\sinh 2u + u\right] + c$$

$$= \frac{25}{4}\left[\sinh u \cosh u + u\right] + c$$

$$= \frac{25}{4}\left[\sinh u\sqrt{1+\sinh^2 u} + u\right] + c$$

$$= \frac{25}{4}\left[\frac{2x}{5}\sqrt{1+\frac{4x^2}{25}} + \operatorname{arsinh}\left(\frac{2x}{5}\right)\right] + c$$

$$= \frac{25}{4}\left[\frac{2x}{5}\sqrt{1+\frac{4x^2}{25}} + \ln\left(\frac{2x}{5} + \sqrt{1+\frac{4x^2}{25}}\right)\right] + c$$