FP2 JANUARY 2009 Q2 COMPLEX NUMBERS

2 i) $|e^{j\frac{\pi}{3}}| = 1 \quad \arg(e^{j\frac{\pi}{3}}) = \frac{\pi}{3}$

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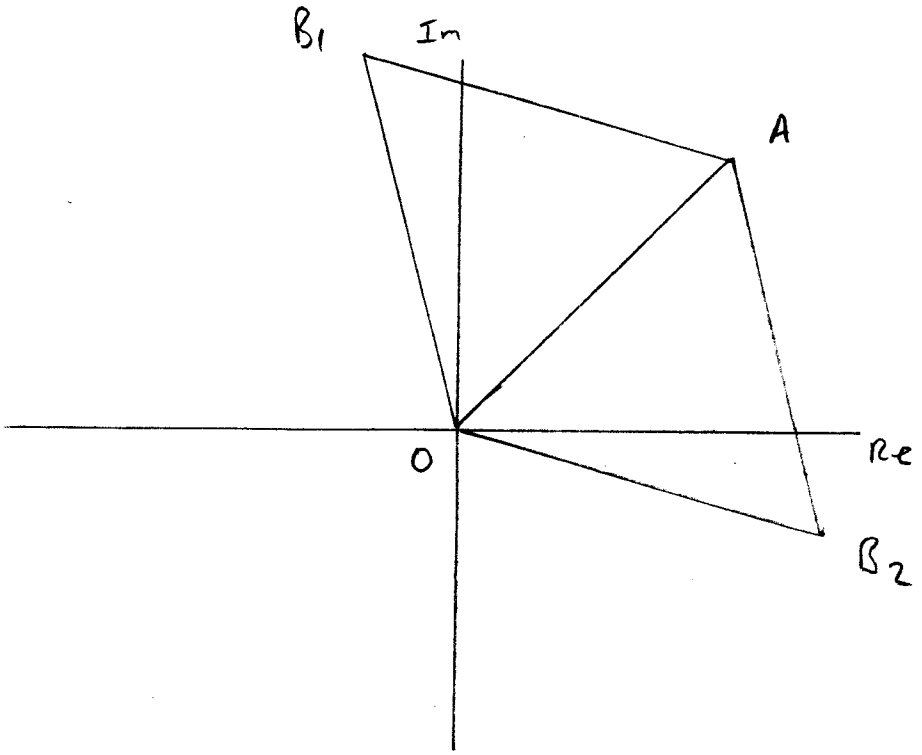
ii) $a = \sqrt{2}(1+j) \quad |a| = \sqrt{2} \times \sqrt{1^2+1^2} = 2$

$\arg(a) = \frac{\pi}{4}$

$\therefore a = 2e^{j\frac{\pi}{4}}$



If b is at B_1 $\quad |b| = 2 \quad \arg(b) = \frac{\pi}{4} + \frac{\pi}{3} = \frac{7\pi}{12}$

$b = 2e^{j\frac{7\pi}{12}}$

If b is at B_2 $\quad |b| = 2 \quad \arg(b) = \frac{\pi}{4} - \frac{\pi}{3} = -\frac{\pi}{12}$

$b = 2e^{-j\frac{\pi}{12}}$

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2 iii)

$$z_1 = \sqrt{2}\, e^{j\frac{\pi}{3}} \Rightarrow z_1^6 = \sqrt{2}^6 e^{j\left(\frac{\pi}{3}\times 6\right)}$$

$$= 8\, e^{j2\pi}$$

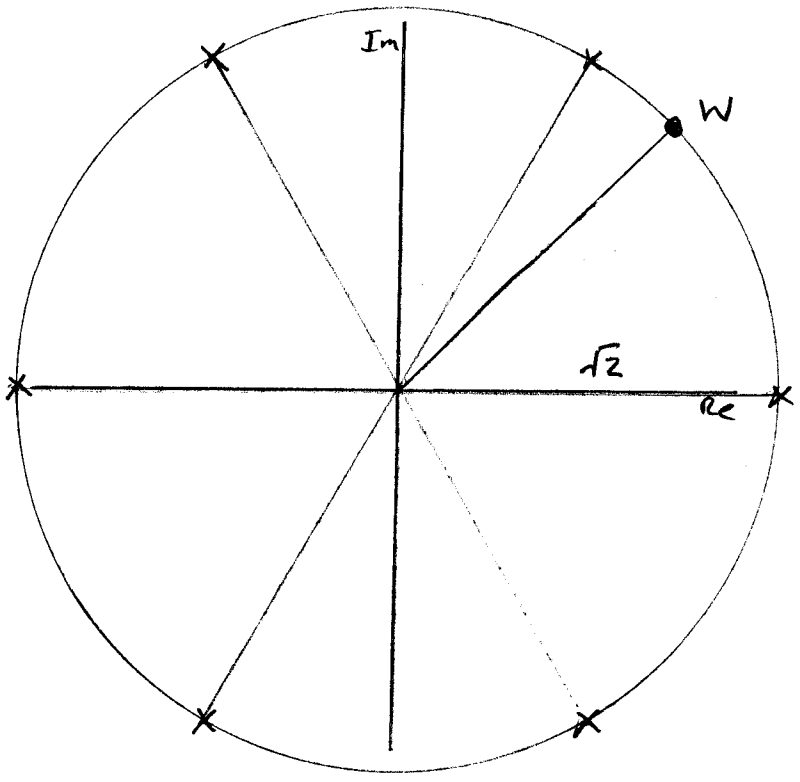
$$= 8(\cos 2\pi + j \sin 2\pi)$$

$$= 8$$

Other sixth roots of 8

Roots given by $\sqrt{2}\, e^{j\left(\frac{\pi}{3} + \frac{2n\pi}{6}\right)}$ for $n = 0, 1, 2, 3, 4, 5$

$\sqrt{2}\, e^{j\frac{2\pi}{3}}$, $\sqrt{2}\, e^{j\pi}$, $\sqrt{2}\, e^{-j\frac{2\pi}{3}}$, $\sqrt{2}\, e^{-j\frac{\pi}{3}}$, $\sqrt{2}\, e^{0}$



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2 iv) $w = z_1 e^{-j\frac{\pi}{12}}$

$\Rightarrow w = \sqrt{2}\, e^{j\frac{\pi}{3}} \times e^{-j\frac{\pi}{12}} = \sqrt{2}\, e^{j\frac{\pi}{4}}$

$w = \sqrt{2}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right)$

$w = \sqrt{2}\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$

$w = 1 + j$

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2 v) $w = \sqrt{2}\, e^{j\frac{\pi}{4}} \Rightarrow w^6 = (\sqrt{2})^6 e^{j\left(\frac{\pi}{4}\times 6\right)}$

$w^6 = 8\, e^{j\frac{3\pi}{2}}$

$w^6 = 8\left(\cos\frac{3\pi}{2} + j\sin\frac{3\pi}{2}\right)$

$w^6 = 8(0 - j)$

$w^6 = -8j$
