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2)
a) i) $z = \cos\theta + j\sin\theta$

$$\Rightarrow \quad z^n + \frac{1}{z^n} = 2\cos(n\theta) \quad , \quad z^n - \frac{1}{z^n} = 2j\sin(n\theta)$$

ii) $$\left(z - \frac{1}{z}\right)^4\left(z + \frac{1}{z}\right)^2 = (2j\sin\theta)^4(2\cos\theta)^2$$

$$= 64\sin^4\theta\cos^2\theta$$

Also $\left(z - \frac{1}{z}\right)^4\left(z + \frac{1}{z}\right)^2$

$$= \left(z^4 - 4z^3\frac{1}{z} + 6z^2\frac{1}{z^2} - 4z\frac{1}{z^3} + \frac{1}{z^4}\right)\left(z^2 + 2z\frac{1}{z} + \frac{1}{z^2}\right)$$

$$= \left(z^4 - 4z^2 + 6 - \frac{4}{z^2} + \frac{1}{z^4}\right)\left(z^2 + 2 + \frac{1}{z^2}\right)$$

$$= z^6 - 4z^4 + 6z^2 - 4 + \frac{1}{z^2}$$
$$+ 2z^4 - 8z^2 + 12 - \frac{8}{z^2} + \frac{2}{z^4}$$
$$+ z^2 - 4 + \frac{6}{z^2} - \frac{4}{z^4} + \frac{1}{z^6}$$

$$= z^6 - 2z^4 - z^2 + 4 - \frac{1}{z^2} - \frac{2}{z^4} + \frac{1}{z^6}$$

$$= \left(z^6 + \frac{1}{z^6}\right) - 2\left(z^4 + \frac{1}{z^4}\right) - \left(z^2 + \frac{1}{z^2}\right) + 4$$

$$= 2\cos 6\theta - 4\cos 4\theta - 2\cos 2\theta + 4$$

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2 a ii) cont)

$$\therefore \quad 64 \sin^4\theta \cos^2\theta = 2\cos 6\theta - 4\cos 4\theta - 2\cos 2\theta + 4$$

$$\Rightarrow \quad \sin^4\theta \cos^2\theta = \frac{1}{32}\cos 6\theta - \frac{1}{16}\cos 4\theta - \frac{1}{32}\cos 2\theta + \frac{1}{16}$$

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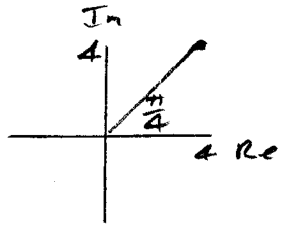
2 b) i) $|4+4j| = \sqrt{4^2+4^2} = \sqrt{32}$ or $4\sqrt{2}$



$$\arg(4+4j) = \frac{\pi}{4}$$

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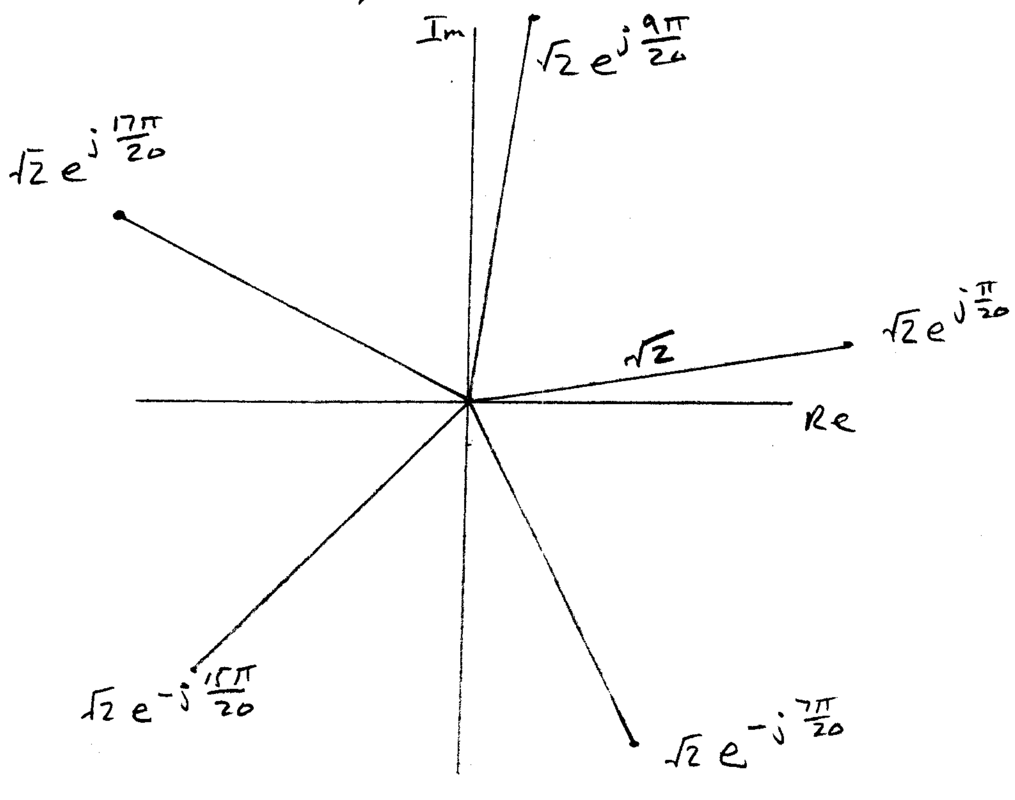
ii) Fifth roots are

$$\left(32^{\frac{1}{2}}\right)^{\frac{1}{5}} e^{j\left(\frac{\pi}{20} + \frac{2n\pi}{5}\right)} \qquad \text{for } n = 0, 1, 2, 3, 4$$

$$= \sqrt{2}\, e^{j\frac{\pi}{20}}, \quad \sqrt{2}\, e^{j\frac{9\pi}{20}}, \quad \sqrt{2}\, e^{j\frac{17\pi}{20}}$$

$$\sqrt{2}\, e^{-j\frac{7\pi}{20}}, \quad \sqrt{2}\, e^{-j\frac{15\pi}{20}}$$



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2b) iii) Find integers p, q such that $(p+qj)^5 = 4+4j$

This will be one of roots already found

$$\sqrt{2}\, e^{-j\frac{15\pi}{20}} = \sqrt{2}\, e^{-j\frac{3\pi}{4}}$$

This would seem most likely with $45°$ angle to horizontal and vertical

$$\sqrt{2}\, e^{-j\frac{3\pi}{4}} = \sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + j\sin\left(-\frac{3\pi}{4}\right)\right)$$

$$= \sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j\right)$$

$$= -1 - 1j$$

$$\Rightarrow \quad p = -1, \quad q = -1$$