FP2 JUNE 2007 Q2 COMPLEX NUMBERS

2a) By de Moivre's theorem

$$(\cos 5\theta + j\sin 5\theta) = (\cos\theta + j\sin\theta)^5 = (c+js)^5$$

where $c = \cos\theta$, $s = \sin\theta$

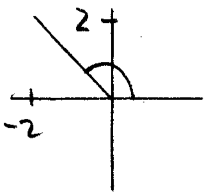
$$(c+js)^5 = c^5 + 5c^4 js + 10c^3 j^2 s^2 + 10c^2 j^3 s^3 + 5c j^4 s^4 + j^5 s^5$$
$$= c^5 + 5c^4 js - 10c^3 s^2 - 10c^2 js^3 + 5cs^4 + js^5$$

Equating imaginary parts

$$\sin 5\theta = 5c^4 s - 10c^2 s^3 + s^5$$
$$= 5(1-s^2)^2 s - 10(1-s^2)s^3 + s^5$$
$$= 5(1-2s^2+s^4)s - 10(s^3 - s^5) + s^5$$
$$= 5s - 10s^3 + 5s^5 - 10s^3 + 10s^5 + s^5$$
$$= 5s - 20s^3 + 16s^5$$
$$= 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$$

2b) $-2+2j$

i)



$$|-2+2j| = \sqrt{8}$$

$$\arg(-2+2j) = \frac{3\pi}{4}$$

$$-2+2j = \sqrt{8}\, e^{j\frac{3\pi}{4}}$$

Cube roots are $(\sqrt{8})^{\frac{1}{3}} e^{\left(\frac{\pi}{4} + \frac{2n\pi}{3}\right)j}$ for $n = 0, 1, 2$

$$= \sqrt{2}\, e^{j\frac{\pi}{4}}, \quad \sqrt{2}\, e^{j\frac{11\pi}{12}}, \quad \sqrt{2}\, e^{-j\frac{5\pi}{12}}$$

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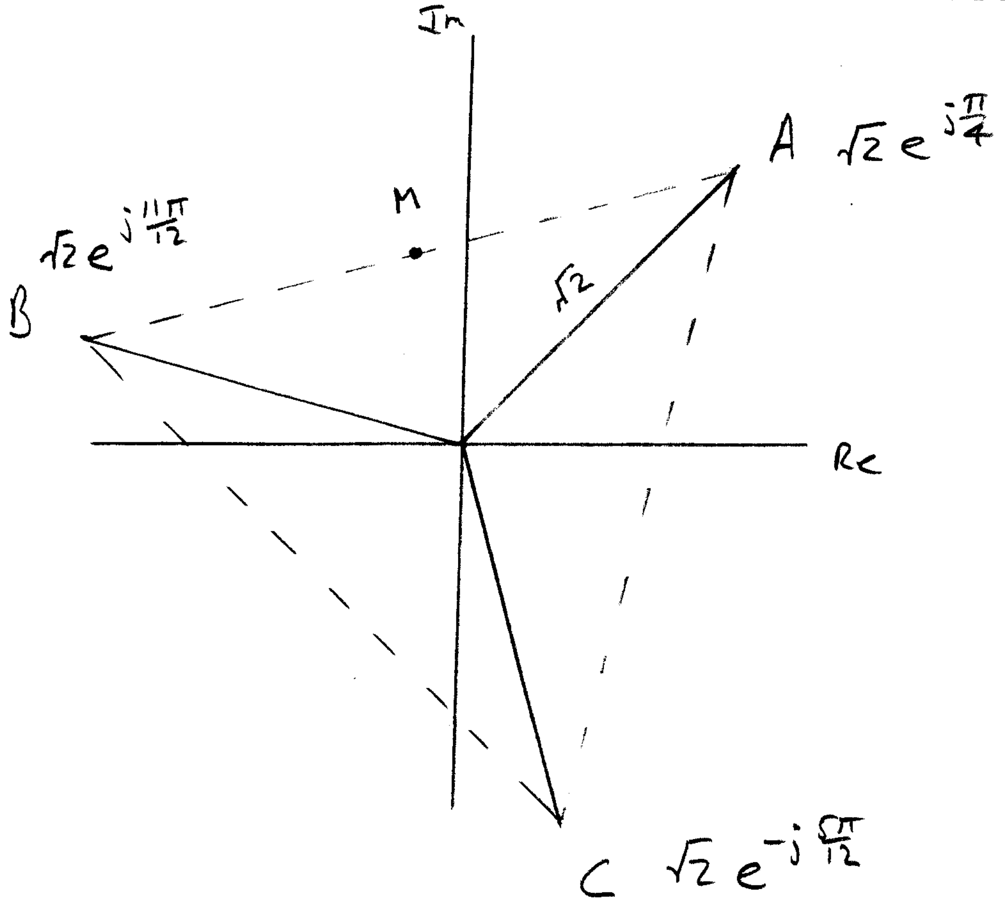
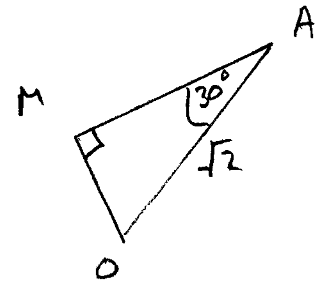
2bii)



2biii)



$$|w| = OM = \sqrt{2}\sin 30 = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\arg(w) = \frac{\pi}{4} + \frac{1}{2}\left(\frac{2\pi}{3}\right)$$

$$= \frac{\pi}{4} + \frac{\pi}{3} = \frac{7\pi}{12}$$

$$|w| = \frac{1}{\sqrt{2}} \qquad \arg(w) = \frac{7\pi}{12}$$

2biv) $w = \frac{1}{\sqrt{2}}e^{j\frac{7\pi}{12}} \Rightarrow w^6 = \left(\frac{1}{\sqrt{2}}\right)^6 e^{j\left(\frac{7\pi}{12}\times 6\right)}$

$$w^6 = \frac{1}{8}e^{j\frac{7\pi}{2}} = \frac{1}{8}e^{-\frac{\pi}{2}}$$

$$w^6 = \frac{1}{8}\left(\cos\left(-\frac{\pi}{2}\right) + j\sin\left(-\frac{\pi}{2}\right)\right) = -\frac{1}{8}j$$