FP2 JUNE 2008 Q4 HYPERBOLIC FUNCTIONS

4 i)

$$\sinh x = \tfrac{1}{2}(e^{x} - e^{-x}) \qquad \cosh x = \tfrac{1}{2}(e^{x} + e^{-x})$$

$$\Rightarrow \cosh^2 x - \sinh^2 x = \tfrac{1}{4}(e^{x}+e^{-x})^2 - \tfrac{1}{4}(e^{x}-e^{-x})^2$$

$$= \tfrac{1}{4}\left[e^{2x} + 2 + e^{-2x} - (e^{2x} - 2 + e^{-2x}\right]$$

$$= \tfrac{1}{4}\left[e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}\right]$$

$$= \tfrac{1}{4}[4] = 1$$

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4 ii)

$$4\cosh^2 x + 9\sinh x = 13$$

$$4(1+\sinh^2 x) + 9\sinh x = 13$$

$$4 + 4\sinh^2 x + 9\sinh x = 13$$

$$4\sinh^2 x + 9\sinh x - 9 = 0$$

$$(4\sinh x - 3)(\sinh x + 3) = 0$$

$$\Rightarrow 4\sinh x - 3 = 0 \quad \text{or} \quad \sinh x + 3 = 0$$

$$4\sinh x = 3 \qquad \sinh x = -3$$

$$\sinh x = \tfrac{3}{4}$$

$$\Rightarrow x = \operatorname{arsinh}\left(\tfrac{3}{4}\right) \quad \text{or} \quad x = \operatorname{arsinh}(-3)$$

$$\Rightarrow x = \ln\left(\tfrac{3}{4} + \sqrt{\left(\tfrac{3}{4}\right)^2 + 1}\right) \qquad x = \ln\left(-3 + \sqrt{(-3)^2+1}\right)$$

$$x = \ln\left(\tfrac{3}{4} + \tfrac{5}{4}\right) \qquad x = \ln\left(-3 + \sqrt{10}\right)$$

$$x = \ln 2$$

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4 iii)

$$y = 4\cosh^2 x + 9\sinh x$$

$$\frac{dy}{dx} = 8\cosh x\sinh x + 9\cosh x$$

At st. pt. $\frac{dy}{dx} = 0$

$$\Rightarrow \quad 8\cosh x\sinh x + 9\cosh x = 0$$

$$\Rightarrow \quad \cosh x(8\sinh x + 9) = 0$$

$\cosh x \neq 0$ since $\cosh x \geq 1$ for all x

$\therefore$ only solution occurs when $8\sinh x + 9 = 0$

$$8\sinh x = -9$$

$$\sinh x = -\frac{9}{8}$$

Find y coord of this st. pt.

$$\cosh^2 x = 1 + \sinh^2 x$$

so when $\sinh x = -\frac{9}{8}$, $\cosh^2 x = 1 + \left(-\frac{9}{8}\right)^2$

$$= 1 + \frac{81}{64} = \frac{145}{64}$$

$$\Rightarrow \quad y = 4\left(\frac{145}{64}\right) + 9\left(-\frac{9}{8}\right)$$

$$y = \frac{145}{16} - \frac{81}{8}$$

$$y = \frac{145 - 162}{16}$$

$$y = -\frac{17}{16}$$

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4 iv)

$$\int_0^{\ln 2} \left(4\cosh^2 x + 9\sinh x\right) dx$$

$$= \int_0^{\ln 2} \left(4\left(\tfrac{1}{2}(e^x+e^{-x})\right)^2 + \tfrac{9}{2}(e^x - e^{-x})\right) dx$$

$$= \int_0^{\ln 2} \left(e^{2x} + 2 + e^{-2x} + \tfrac{9}{2}e^x - \tfrac{9}{2}e^{-x}\right) dx$$

$$= \left[\tfrac{1}{2}e^{2x} + 2x - \tfrac{1}{2}e^{-2x} + \tfrac{9}{2}e^x + \tfrac{9}{2}e^{-x}\right]_0^{\ln 2}$$

$$= \left(\tfrac{1}{2}e^{2\ln 2} + 2\ln 2 - \tfrac{1}{2}e^{-2\ln 2} + \tfrac{9}{2}e^{\ln 2} + \tfrac{9}{2}e^{-\ln 2}\right) - \left(\tfrac{1}{2} - \tfrac{1}{2} + \tfrac{9}{2} + \tfrac{9}{2}\right)$$

$$= \left(\tfrac{1}{2}e^{\ln 4} + 2\ln 2 - \tfrac{1}{2}^{\ln(\frac{1}{4})} + \tfrac{9}{2}\times 2 + \tfrac{9}{2}\times\tfrac{1}{2}\right) - (9)$$

$$= 2 + 2\ln 2 - \tfrac{1}{8} + 9 + \tfrac{9}{4} - 9$$

$$= 2\ln 2 + \tfrac{33}{8}$$

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4 iv)

Alternative method

$$\int_0^{\ln 2} \left(4\cosh^2 x + 9\sinh x\right) dx$$

$$= \int_0^{\ln 2} \left(2(1+\cosh 2x) + 9\sinh x\right) dx$$

$$= \int_0^{\ln 2} \left(2 + 2\cosh 2x + 9\sinh x\right) dx$$

$$= \left[2x + \sinh 2x + 9\cosh x\right]_0^{\ln 2}$$

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4iv cont)

$$\left(2\ln 2 + \tfrac{1}{2}\left(e^{2\ln 2} - e^{-2\ln 2}\right) + \tfrac{9}{2}\left(e^{\ln 2} + e^{-\ln 2}\right)\right)$$

$$-\left(0 + 0 + 9\right)$$

$$= 2\ln 2 + \tfrac{1}{2}\left(e^{\ln 4} - e^{-\ln 4}\right) + \tfrac{9}{2}\left(2 + \tfrac{1}{2}\right) - 9$$

$$= 2\ln 2 + \tfrac{1}{2}\left(4 - \tfrac{1}{4}\right) + 9 + \tfrac{9}{4} - 9$$

$$= 2\ln 2 + 2 - \tfrac{1}{8} + 9 + \tfrac{9}{4} - 9$$

$$= 2\ln 2 + \tfrac{33}{8}$$