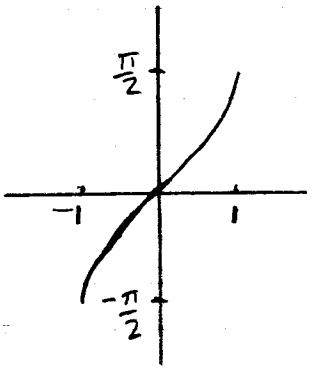
FP2 JUNE 2009 Q3 MAINLY COMPLEX NUMBERS

3 a) i) $y = \arcsin x$



$y = \arcsin x$

$\Rightarrow \quad \sin y = x$

$\Rightarrow \quad \cos y \frac{dy}{dx} = 1$

$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\cos y}$

$\Rightarrow \quad \frac{dy}{dx} = \pm \frac{1}{\sqrt{1-\sin^2 y}}$

$\Rightarrow \quad \frac{dy}{dx} = \pm \frac{1}{\sqrt{1-x^2}}$

From graph, $\frac{dy}{dx} > 0$

$\therefore \quad \frac{dy}{dx} = + \frac{1}{\sqrt{1-x^2}}$

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ii) $\int_0^1 \frac{1}{\sqrt{2-x^2}} = \int_0^1 \frac{1}{\sqrt{\sqrt{2}^2 - x^2}} dx$

$= \left[\arcsin\left(\frac{x}{\sqrt{2}}\right) \right]_0^1$

$= \arcsin \frac{1}{\sqrt{2}} - \arcsin 0$

$= \frac{\pi}{4} - 0 \qquad = \frac{\pi}{4}$

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3b) $C = \cos\theta + \frac{1}{3}\cos 3\theta + \frac{1}{9}\cos 5\theta + \ldots$

$S = \sin\theta + \frac{1}{3}\sin 3\theta + \frac{1}{9}\cos 5\theta + \ldots$

$C+jS = (\cos\theta + j\sin\theta) + \frac{1}{3}(\cos 3\theta + j\sin 3\theta) + \frac{1}{9}(\cos 5\theta + j\sin 5\theta) + \ldots$

$C+jS = e^{j\theta} + \frac{1}{3}e^{j3\theta} + \frac{1}{9}e^{j5\theta} + \ldots$

This is a GP with first term $a = e^{j\theta}$

common ratio $r = \frac{1}{3}e^{j2\theta}$

Sum to infinity exists since $|\frac{1}{3}e^{j2\theta}| = \frac{1}{3} < 1$

$$S_\infty = \frac{a}{1-r}$$

$$\Rightarrow \quad C+jS = \frac{e^{j\theta}}{1-\frac{1}{3}e^{j2\theta}}$$

$$= \frac{e^{j\theta}\left(1-\frac{1}{3}e^{-j2\theta}\right)}{\left(1-\frac{1}{3}e^{j2\theta}\right)\left(1-\frac{1}{3}e^{-j2\theta}\right)}$$

$$C+jS = \frac{e^{j\theta} - \frac{1}{3}e^{-j\theta}}{1-\frac{1}{3}\left(e^{j2\theta}+e^{-j2\theta}\right)+\frac{1}{9}}$$

$$C+jS = \frac{\cos\theta + j\sin\theta - \frac{1}{3}(\cos\theta - j\sin\theta)}{\frac{10}{9} - \frac{2}{3}\cos 2\theta}$$

$$C+jS = \frac{9\cos\theta + 9j\sin\theta - 3\cos\theta + 3j\sin\theta}{10 - 6\cos 2\theta}$$

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3b cont)

$$C + jS = \frac{6\cos\theta + 12j\sin\theta}{10 - 6\cos 2\theta}$$

$$C + jS = \frac{3\cos\theta + 6j\sin\theta}{5 - 3\cos 2\theta}$$

Equating real and imaginary parts

$$C = \frac{3\cos\theta}{5 - 3\cos 2\theta}$$

$$S = \frac{6\sin\theta}{5 - 3\cos\theta}$$