MEI MECHANICS 1 JAN 2005 NEW SYLLABUS

1) $\underline{r} = t^2\underline{i} + (5t - 2t^2)\underline{j}$

i) $\dot{\underline{r}} = 2t\underline{i} + (5 - 4t)\underline{j}$

$\underline{a} = \ddot{\underline{r}} = 2\underline{i} - 4\underline{j}$

Using $\underline{F} = m\underline{a}$

$\underline{F} + 12\underline{j} = 4(2\underline{i} - 4\underline{j})$

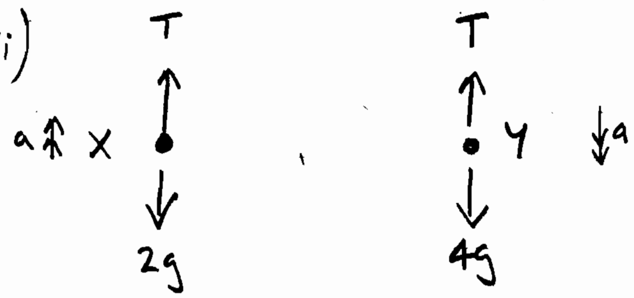
$\underline{F} = 8\underline{i} - 16\underline{j} - 12\underline{j}$

$\underline{F} = 8\underline{i} - 28\underline{j}$

2) i) A) pulleys are smooth and string is light

B) string is inextensible

ii)



iii) For X

$T - 2g = 2a$ ①

For Y $4g - T = 4a$ ②

Adding ① + ② gives

$2g = 6a$

$\Rightarrow a = \dfrac{g}{3} = \dfrac{9.8}{3}$

$a = 3.27\ \text{ms}^{-2}$ to 3 s.f.

Subst in ①

$T - 2g = \dfrac{2g}{3}$

$T = \dfrac{2g}{3} + 2g = \dfrac{8g}{3}$

$T = 26.1\ \text{N}$ to 3 s.f.

3) $\begin{pmatrix} x \\ -7 \\ z \end{pmatrix} + \begin{pmatrix} 4 \\ y \\ -5 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

i) $x = -9\ \text{N}$

$y = 3\ \text{N}$

$z = 12\ \text{N}$

ii) $\left\| \begin{pmatrix} 5 \\ 4 \\ -7 \end{pmatrix} \right\| = \sqrt{5^2 + 4^2 + (-7)^2}$

$= \sqrt{90}\ \text{N}$

4) i) Using $v^2 = u^2 + 2as$

At peak $v = 0$

$0 = 21^2 - 2 \times 9.8 \times s$

$19.6s = 21^2$

$s = \dfrac{21^2}{19.6} = 22.5\ \text{m}$

4 ii) Find distance fallen by original particle in 1.5 s

Using $s = ut + \frac{1}{2}at^2$

$$s_1 = 0 + \frac{1}{2} \times 9.8 \times 1.5^2$$

$$s_1 = 11.025 \text{ m}$$

Original particle falls 11.025 m

Find rise of 2nd particle

$$s_2 = ut + \frac{1}{2}at^2$$

$$s_2 = 15 \times 1.5 - \frac{1}{2} \times 9.8 \times 1.5^2$$

$$s_2 = 11.475 \text{ m}$$

Particles collide after 1.5 s

Since $11.475 + 11.025$

$= 22.5\text{m}$

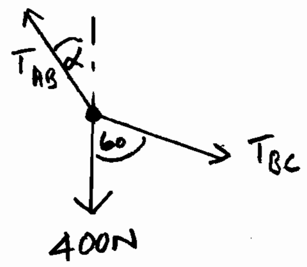
the original distance apart when 2nd particle launched

Height of collision above O

$= 11.475 \text{ m}$

5)

i)



ii) Lami's Theorem

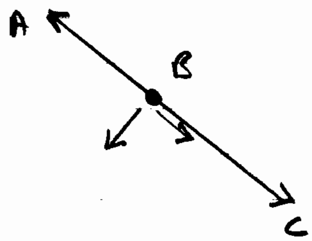
$$\frac{T_{AB}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{400}{\sin 150^\circ}$$

$$\Rightarrow T_{BC} = 400 \text{ N}$$

$$\Rightarrow T_{AB} = \frac{400}{\sin 150^\circ} \times \sin 60^\circ$$

$$T_{AB} = 693 \text{ N} \quad \text{to 3 s.f.}$$

iii) If $\alpha = 60^\circ$ and BC remains at 60° to vertical then ABC would be a straight line



Resolving the weight of the box parallel and perpendicular to ABC, the perpendicular component could not be supported by the two strings. $\therefore$ an impossible situation.

6) i) Distance fallen represented by area under graph

Area of Δ from $t=0$ to $t=2$

$$= \frac{1}{2} \times 2 \times 20 = 20\text{m}$$

Area of trapezium from $t=2$ to $t=6$

$$= \frac{1}{2}(20+10) \times 4 = 60\text{m}$$

Area of Δ from $t=6$ to $t=7$

$$= \frac{1}{2} \times 1 \times 10 = 5\text{m}$$

Distance fallen $= 20+60+5$

$= 85\text{m}$

ii)

$$a = \frac{v-u}{t} = \frac{10-20}{4}$$

$$a = -2.5 \text{ ms}^{-2}$$

Ball is slowing down as it falls $\therefore$ acceleration during this period is 2.5 ms^{-2} vertically upwards

iii) Using $v = u + at$

$$v = 20 - 2.5(t-2)$$

$$v = 20 - 2.5t + 5$$

$$v = 25 - 2.5t \text{ ms}^{-1}$$

iv) g has been taken as 10 ms^{-2} and air resistance has been assumed to be negligible

v)

$$v = -\frac{3}{2}t^2 + \frac{19}{2}t + 7$$

When $t=2$, $v = -\frac{3}{2}(2)^2 + \frac{19}{2}(2) + 7$

$= 20\text{ ms}^{-1}$ ✓

when $t=6$, $v = -\frac{3}{2}(6)^2 + \frac{19}{2}(6) + 7$

$= 10\text{ ms}^{-1}$ ✓

when $t=7$ $v = -\frac{3}{2}(7)^2 + \frac{19}{2}(7) + 7$

$= 0\text{ ms}^{-1}$ ✓

vi) Distance fallen from $t=2$ to $t=7$ is given by

$$\int_2^7 \left(-\frac{3}{2}t^2 + \frac{19t}{2} + 7\right) dt$$

$$= \left[-\frac{t^3}{2} + \frac{19t^2}{4} + 7t\right]_2^7$$

$$= \left(-\frac{343}{2} + \frac{931}{4} + 49\right) - \left(-\frac{8}{2} + \frac{76}{4} + 14\right)$$

$= 110.25 - 29$

$= 81.25\text{ m}$

7) i) Horizontal motion

$x = u\cos\alpha \times t$

$x = 40\cos 50^\circ \times t$

Vertical motion

$y = u\sin\alpha\, t - \frac{1}{2}gt^2$

$y = 40\sin 50^\circ \times t - 4.9t^2$

ii) Lands at D when $y = 0$

$\Rightarrow \quad 40\sin 50^\circ \times t - 4.9t^2 = 0$

$t(40\sin 50^\circ - 4.9t) = 0$

Either $t = 0$ (Point A)

or $t = \dfrac{40\sin 50^\circ}{4.9} = 6.253\text{ s}$ (Point D)

Range of stone

$= u\cos\alpha \times t$

$= 40\cos 50^\circ \times 6.253$

$= 160.774\text{ m}$

$= 160.8\text{ m}$ to 4 s.f.

iii) Horizontal motion

$\text{time} = \dfrac{\text{distance}}{\text{speed}} = \dfrac{30}{40\cos 50^\circ}$

$t = 1.167\text{ s}$

Time from A to B will equal time from C to D

$\therefore$ time from A to C

$= 6.253 - 1.167$

$= 5.086\text{ s}$

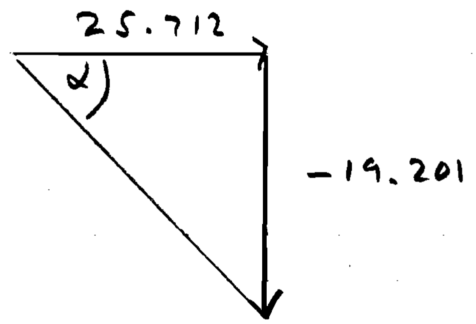
iv) At C

$V_x = 40\cos 50^\circ = 25.712\text{ ms}^{-1}$

$V_y = u_y + at$

$V_y = 40\sin 50^\circ - 9.8 \times 5.086$

$V_y = -19.201\text{ ms}^{-1}$



$\alpha = \tan^{-1}\left(\dfrac{19.201}{25.712}\right)$

$\alpha = 36.75^\circ$

Direction 36.75° below horizontal