1) i)

At $t = 4\,s$ $a = 8\,ms^{-2}$

Change in speed represented by area under graph

$= \frac{1}{2} \times 4 \times 8 = 16\,ms^{-1}$

Since it starts from rest

speed $= 16\,ms^{-1}$

ii)

$a = 2t$

iii)

Speed greatest at $t = 7$ since acceleration > 0 until then. Thereafter $a < 0$ so particle is slowing down

iv)

Change in speed from $t = 5$ to $t = 7$

$= \frac{1}{2} \times 2 \times 8 = 8\,ms^{-1}$

Change in speed from

$t = 7$ to $t = 8$

$= \frac{1}{2} \times 1 \times -4$

$= -2\,ms^{-1}$

Overall change $= 8 - 2 = 6\,ms^{-1}$

An increase by $6\,ms^{-1}$

2)

$v = 24t - 6t^2$

i)

$a = \frac{dv}{dt} = 24 - 12t$ ms^{-2}

ii)

$v = 24t - 6t^2$

$v = 2t(12 - 3t)$

$v = 0 \Rightarrow t = 0\,s$ or $t = 4\,s$

$t_1 = 0\,s$, $t_2 = 4\,s$

iii)

Distance $= \int_0^4 (24t - 6t^2)\,dt$

$= \left[12t^2 - 2t^3\right]_0^4$

$= (12 \times 16 - 2 \times 64) - (0 - 0)$

$= 64\,m$

3)

i)

$\begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 21 \\ -7 \end{pmatrix} + \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow R_1 = -18$, $R_2 = 3$

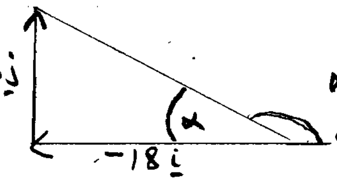
$\underline{R} = -18\underline{i} + 3\underline{j}$

ii)

$|\underline{R}| = \sqrt{(-18)^2 + 3^2}$

$= \sqrt{333} = 18.2\,N$

to 3 s.f.



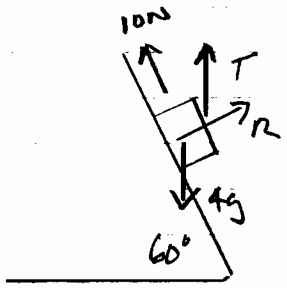
$\alpha = \tan^{-1}\left(\frac{3}{18}\right)$

$\alpha = 9.5°$

$\underline{R}$ makes angle of $170.5°$ with $\underline{i}$ direction

4)

i)



ii) Parallel to slope

$$10 + T\sin 60° = 4g\sin 60°$$

$$T = \frac{4g\sin 60° - 10}{\sin 60°}$$

$$T = 27.7 \text{ N}$$

iii) $\perp$ to slope

$$4g\cos 60° = T\cos 60 + R$$

$$R = 4g\cos 60° - T\cos 60°$$

$$R = 5.77 \text{ N}$$

5)

i)

$$\underline{r} = \tfrac{1}{2}t\,\underline{i} + (t^2-1)\,\underline{j}$$

$$\underline{r} = \begin{pmatrix} \frac{t}{2} \\ t^2-1 \end{pmatrix}$$

When $x = 2$, $t = 4$

When $t = 4$, $y = 4^2 - 1 = 15$

ii)

$$x = \frac{t}{2}, \quad y = t^2 - 1$$

$$\Rightarrow t = 2x$$

Substitute for t

$$y = (2x)^2 - 1$$

$$y = 4x^2 - 1$$

iii) If moving at 45° to Ox and Oy

then $\frac{dy}{dx} = 1$

But $\frac{dy}{dx} = 8x$

$$\therefore \quad 8x = 1$$

$$x = \frac{1}{8}$$

Subst for x

$$y = 4 \times \left(\tfrac{1}{8}\right)^2 - 1$$

$$y = \frac{4}{64} - 1$$

$$y = -\frac{15}{16}$$

Moving at 45° to Ox and Oy at point $\left(\frac{1}{8}, -\frac{15}{16}\right)$

6) i)

$$F = ma$$

$$2000 = 1000a$$

$$\Rightarrow a = 2\ \text{ms}^{-2}$$

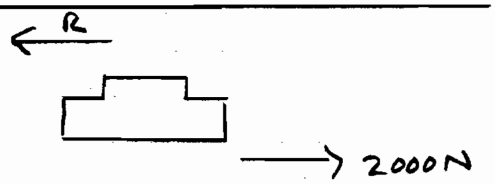
$$a = \frac{v-u}{t}$$

$$\Rightarrow t = \frac{v-u}{a}$$

$$t = \frac{12.5-5}{2}$$

$$t = 3.75\ \text{s}$$

ii)



$$2000 - R = ma$$

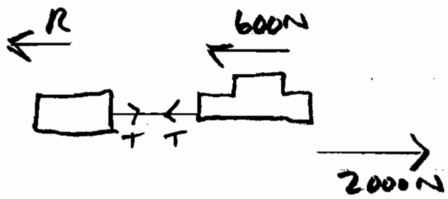
$$2000 - R = 1000 \times 1.4$$

$$2000 - R = 1400$$

$$2000 - 1400 = R$$

$$R = 600\ \text{N}$$

iii)



For whole system $F = ma$

$$2000 - 600 - R = 1800 \times 0.7$$

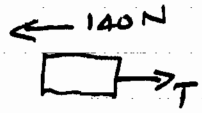
$$1400 - R = 1260$$

$$1400 - 1260 = R$$

$$R = 140\ \text{N}$$

iv)

For trailer



$$F = ma$$

$$T - 140 = 800 \times 0.7$$

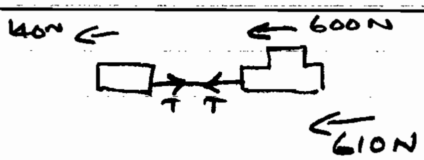
$$T - 140 = 560$$

$$T = 560 + 140$$

$$T = 700\ \text{N}$$

Tension in towbar = 700 N

v)

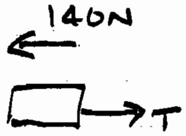


For whole system $F = ma$

$$-610 - 600 - 140 = 1800a$$

$$a = -0.75\ \text{ms}^{-2}$$

For trailer



$$F = ma$$

$$T - 140 = 800 \times (-0.75)$$

$$T = -460\ \text{N}$$

Towbar is under a compression of 460 N

7)

i)

$u = \sqrt{10^2 + 12^2} = 15.6\ ms^{-1}$

$\theta = \tan^{-1}\left(\frac{12}{10}\right) = 50.2^\circ$

ii)

Using $s = ut + \frac{1}{2}at^2 + s_0$

$y = 12t - \frac{1}{2}\times 10t^2 + 9$

$y = 9 + 12t - 5t^2$

where y is height above ground

At time t the horizontal distance x from O is

$x = 10t$

iii)

Defining launch point P as new orgin and using $v^2 = u^2 + 2as$

At max height above point of projection $v = 0$

$\Rightarrow\ 0 = 12^2 - 2\times 10y$

$0 = 144 - 20y$

$20y = 144$

$y = 7.2\ m$

Max height above point of projection $= 7.2\ m$

iv)

At X, $y = -9\ m$

Using $s = ut + \frac{1}{2}at^2$

$-9 = 12t - 5t^2$

$5t^2 - 12t - 9 = 0$

$(5t+3)(t-3) = 0$

$\Rightarrow\ t = 3$ or $\cancel{t = -0.6}$

when $t = 3$, $x = 10\times 3 = 30m$

$\therefore\ OX = 30m$

v)

For particle B $u_x = 20\cos 60^\circ = 10\ ms^{-1}$

$\therefore\ x = 10t$ as for A

vi)

Using $s = ut + \frac{1}{2}at^2$

$y = 20\sin 60^\circ t - \frac{1}{2}\times 10t^2$

$y = 20\times\frac{\sqrt{3}}{2}t - 5t^2$

$y = 10\sqrt{3}t - 5t^2$

vii)

Collide when $y_A = y_B$

$\Rightarrow\ 9 + 12t \cancel{-5t^2} = 10\sqrt{3}t \cancel{-5t^2}$

$9 = 10\sqrt{3}t - 12t$

$9 = (10\sqrt{3} - 12)t$

$t = \frac{9}{(10\sqrt{3}-12)} = 1.69s \approx 1.7s$