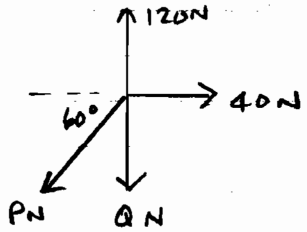
1) i)



Resolve horizontally

$P\cos 60° = 40$

$\Rightarrow P = \dfrac{40}{\cos 60°} = 80\text{ N}$

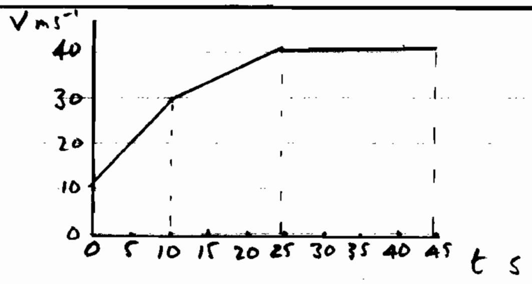
1ii) Resolving vertically

$120 = P\sin 30 + Q$

$Q = 120 - 80\sin 60°$

$Q = 50.7\text{ N}$ to 3 s.f.

2) i)



2ii)

Area under graph represents distance AB. Area =

$\frac{1}{2}(10+30)\times 10 + \frac{1}{2}(30+40)\times 15 + 40\times 20$

Distance AB $= 1525$ m

2iii)

Extra distance $= 1700 - 1525$
$= 175$ m

Using $s = \left(\dfrac{u+v}{2}\right)\times t$

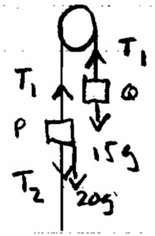
$175 = \left(\dfrac{40+0}{2}\right)\times T$

$T = \dfrac{2\times 175}{40} = 8.75$ s

3) i)

Because the pulley is smooth and the string is light

3ii)



Resolving vertically at Q

$T_1 = 15g$

Resolving at P

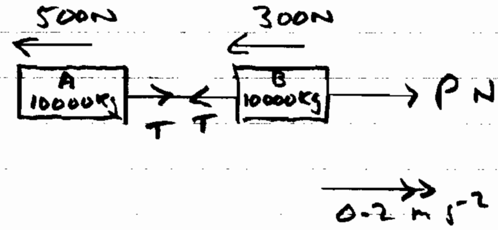
$T_2 + 20g = T_1$

$\therefore T_2 = 15g - 20g = -5g$

$T_2 = -49\text{ N}$

Force in rod is a 49N thrust

4) i)



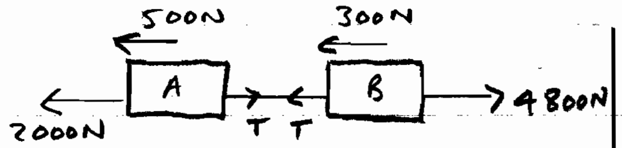
For whole system $F = ma$

$P - 500 - 300 = 20000\times 0.2$

$P - 800 = 4000$

$P = 4800\text{ N}$

4ii)



For whole system $F = ma$

$$4800 - 2000 - 500 - 300 = 20000a$$

$$2000 = 20000a$$

$$\Rightarrow \quad a = 0.1 \text{ ms}^{-2}$$

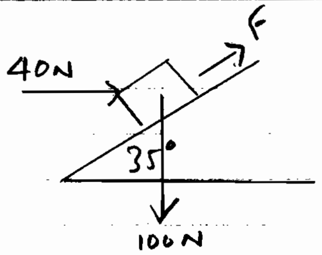
4iii) For truck B $\quad F = ma$

$$4800 - 300 - T = 10000 \times 0.1$$

$$4500 - T = 1000$$

$$T = 3500 \text{ N}$$

5)



Resolve parallel to slope

$$40\cos 35° + F = 100\sin 35°$$

$$F = 100\sin 35° - 40\cos 35°$$

$F = 24.6$ N to 3 s.f.

Direction is up the slope

6) i) Resultant $\underline{F} = \begin{pmatrix} 0 \\ 0 \\ -80 \end{pmatrix} + \begin{pmatrix} -1 \\ 16 \\ 72 \end{pmatrix}$

$$\underline{F} = \begin{pmatrix} -1 \\ 16 \\ -8 \end{pmatrix}$$

Using $\underline{F} = m\underline{a}$

$$\begin{pmatrix} -1 \\ 16 \\ -8 \end{pmatrix} = 8\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$\Rightarrow \quad \underline{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{8} \\ 2 \\ -1 \end{pmatrix}$$

$$\underline{a} = -\tfrac{1}{8}\underline{i} + 2\underline{j} - \underline{k}$$

6ii)

$$\underline{v} = \int \underline{a}\, dt$$

$$\underline{v} = \int \begin{pmatrix} -\frac{1}{8} \\ 2 \\ -1 \end{pmatrix} dt$$

$$\underline{v} = \begin{pmatrix} -\frac{1}{8}t + C \\ 2t + D \\ -t + E \end{pmatrix}$$

When $t = 0$ $\quad \underline{v} = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$

$$\therefore \quad \underline{v} = \begin{pmatrix} -\frac{1}{8}t + 1 \\ 2t - 4 \\ -t + 3 \end{pmatrix}$$

$$\underline{r} = \int \underline{v}\, dt$$

$$\underline{r} = \begin{pmatrix} -\frac{1}{16}t^2 + t + A \\ t^2 - 4t + B \\ -\frac{t^2}{2} + 3t + C \end{pmatrix}$$

$\underline{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ when $t = 0$

$$\therefore \quad A = B = C = 0$$

6 ii) cont)
$$\underline{r} = \begin{pmatrix} -\frac{1}{16}t^2 + t \\ t^2 - 4t \\ -\frac{t^2}{2} + 3t \end{pmatrix}$$

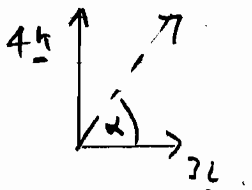
At A when $t = 4$

$$\underline{r}_A = \begin{pmatrix} -\frac{16}{16} + 4 \\ 16 - 16 \\ -\frac{16}{2} + 12 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$$

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} = 3\underline{i} + 4\underline{k}$$

6 iii)
$$|\overrightarrow{OA}| = \sqrt{3^2 + 0^2 + 4^2} = 5\text{ m}$$

6 iv)



Angle with horizontal

$$= \alpha = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$$

7 i)
$$v = t^2 - 2t - 8$$

At $t = 0$, $v = -8\text{ ms}^{-1}$

7 ii)
When $t = -2$,
$$v = (-2)^2 - 2(-2) - 8$$
$$= 4 + 4 - 8$$
$$= 0\text{ ms}^{-1}$$

When $t = 4$,
$$v = 4^2 - 2(4) - 8$$
$$= 16 - 8 - 8$$
$$= 0\text{ ms}^{-1}$$

7 iii)
$$a = \frac{dv}{dt} = 2t - 2$$

When $a = 0$, $2t - 2 = 0$
$$\Rightarrow t = 1$$

When $t = 1$
$$v = 1^2 - 2(1) - 8$$
$$v = -9\text{ ms}^{-1}$$

7 iii cont)
A is point $(1, -9)$

7 iv)
$$s = \int_1^4 v\,dt = \int_1^4 (t^2 - 2t - 8)\,dt$$

$$s = \left[\frac{t^3}{3} - t^2 - 8t\right]_1^4$$

$$= \left(\frac{64}{3} - 16 - 32\right) - \left(\frac{1}{3} - 1 - 8\right)$$

$$= \left(-26\tfrac{2}{3}\right) - \left(-8\tfrac{2}{3}\right)$$

$$= -18\text{ m}$$

Which is 18 m in negative direction

7 v)
$2 \times 18 = 36$ m
in negative direction

(due to symmetry of graph)

7 vi)
From $t = 4$ to $t = 5$

Walks $\left[\frac{t^3}{3} - t^2 - 8t\right]_4^5$

$$= \left(\frac{125}{3} - 25 - 40\right) - \left(-26\tfrac{2}{3}\right)$$

$$= -23\tfrac{1}{3} + 26\tfrac{2}{3} = 3\tfrac{1}{3}\text{ m}$$

7vi) cont) Total distance walked

$= 18 + 3\frac{1}{3} = 21\frac{1}{3}$ m

(The walks between $t=1$ and $t=4$ and between $t=4$ and $t=5$ must be calculated separately because they are in opposite directions and a single integral from $t=1$ to $t=5$ will give the displacement rather than the total distance walked. In effect, areas above and below the t axis on the graph cancel out.)

8) i)

Vertically using $s = ut + \frac{1}{2}at^2$

$y = u\sin\alpha \times t - \frac{1}{2}gt^2$

$y = 25 \times 0.28 \times t - 4.9t^2$

$y = 7t - 4.9t^2$

Horizontally $x = u\cos\alpha \times t$

$x = 25 \times 0.96 \times t$

$x = 24t$

8ii) Using $v^2 = u^2 + 2as$

$v_y^2 = u_y^2 - 19.6y$

At max height $v_y = 0$

$\Rightarrow \quad 0 = 7^2 - 19.6y$

$y = \frac{49}{19.6} = 2.5$ m

Max height $= 2.5$ m

8iii) Using $y = 7t - 4.9t^2$

Half max height when $y = 1.25$ m

$1.25 = 7t - 4.9t^2$

$4.9t^2 - 7t + 1.25 = 0$

By calculator $t = 1.2193$ s and t 0.2092 s

time interval $= 1.2193 - 0.2092$
$= 1.0101$ s

Horizontal distance travelled

$x = 24 \times 1.0101$

$x = 24.2424$

$x = 24.2$ m to 3 s.f.

8iv) A) when $t = 1.25$

Using $v = u + at$

$V_y = 7 - 9.8t$

$V_y = 7 - 9.8 \times 1.25 = -5.25\ \text{ms}^{-1}$

B) Falling since $V_y < 0$

c) $V_y = -5.25\ \text{ms}^{-1}$ $V_x = U_x = 24\ \text{ms}^{-1}$

Speed $= \sqrt{(-5.25)^2 + 24^2}$

$= 24.6\ \text{ms}^{-1}$ to 3 s.f.

8v) $y = 7t - 4.9t^2$ (1)

$x = 24t$ (2)

from (2) $t = \frac{x}{24}$

Subst for t in (1)

$$y = 7\frac{x}{24} - 4.9\frac{x^2}{24^2}$$

$$y = \frac{7x}{24} - \frac{4.9x^2}{576}$$

$$y = \frac{7 \times 24x - 4.9x^2}{576}$$

$$y = \frac{0.7x}{576}(240 - 7x)$$

Ball lands when $y = 0$

$\Rightarrow x = 0$ or $240 - 7x = 0$

$7x = 240$

$x = 34.3$

Range $= 34.3$ m to 3 s.f.