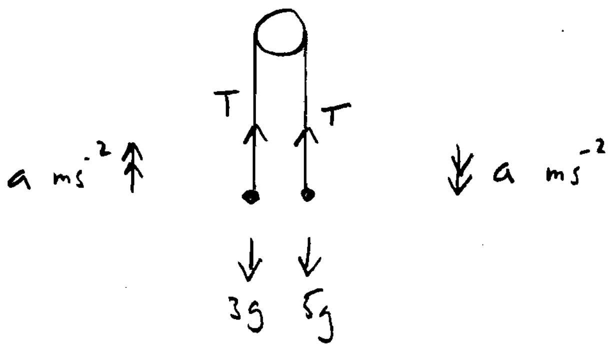
MEI MECHANICS 1 JANUARY 2012

1)



$F = ma$ for each block

$T - 3g = 3a$ ①

$5g - T = 5a$ ②

①+②

$2g = 8a$

$\Rightarrow a = \frac{2g}{8} = \frac{2 \times 9.8}{8}$

$\underline{a = 2.45\ ms^{-2}}$

Sub for a in ①

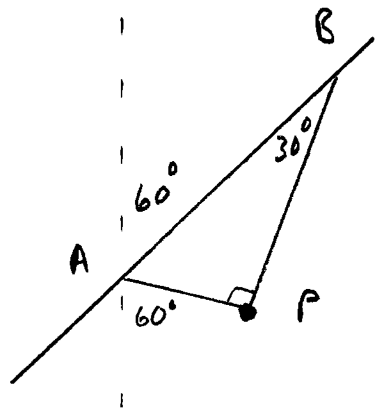
$T - 3g = 3 \times 2.45$

$T = 3 \times 2.45 + 3 \times 9.8$

$\underline{T = 36.8\ N}$

2)

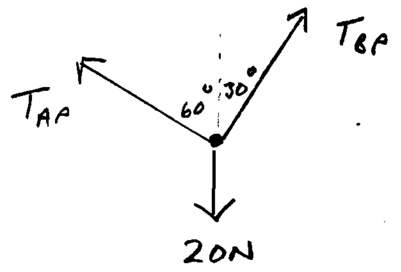
i)



$\angle BAP = 60°$ ($\angle$s on straight line)

$\therefore \angle APB = 90°$ ($\angle$ sum of Δ)

ii)



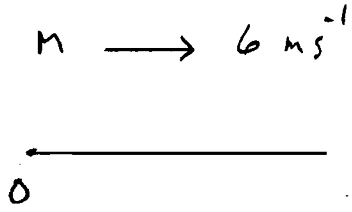
iii Lami's Theorem

$$\frac{T_{AP}}{\sin 150°} = \frac{T_{BP}}{\sin 120°} = \frac{20}{\sin 90}$$

$T_{AP} = 20 \sin 150 = 10\ N$

$T_{BP} = 20 \sin 120 = 17.3\ N$

3)



Nina $a = 4 - t$

i) For Nina $v = \int a\, dt$

$= \int (4 - t)\, dt$

$v = 4t - \frac{t^2}{2} + c$

$v = 0$ when $t = 0 \Rightarrow c = 0$

so $v = 4t - \frac{t^2}{2}$ for $0 \le t \le 4$

MEI MECHANICS 1 JANUARY 2012

When $t = 4$

$$v = 4\times 4 - \frac{4^2}{2} = 16-8 = 8$$

Since $a = 0$ for $t > 4$

$v = 8\,ms^{-1}$ for $t > 4$

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ii) $$s = \int_0^4 v\,dt$$

$$s = \int_0^4 \left(4t - \frac{t^2}{2}\right) dt$$

$$s = \left[2t^2 - \frac{t^3}{6}\right]_0^4$$

$$s = \left(2\times 4^2 - \frac{4^3}{6}\right) - (0-0)$$

$$s = 32 - \frac{64}{6}$$

$$s = 21\tfrac{1}{3}\text{ m}$$

When $t = 5\frac{1}{3}$

$s = 21\frac{1}{3} + 1\frac{1}{3}\times 8$

= distance in 4 s + distance in $1\frac{1}{3}$ s

$= 21\frac{1}{3} + 10\frac{2}{3}$

$= 32$m

After $5\frac{1}{3}$ s Marie has run $5\frac{1}{3}\times 6 = 32$m past O

$\therefore$ both are 32m past O at that time $t = 5\frac{1}{3}$ s

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4) $$\underline{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 30 \\ 40 \end{pmatrix} t - \begin{pmatrix} 0 \\ 5 \end{pmatrix} t^2$$

i) A) Projected from height $y = 5$m

B) g taken as 10 (from the $-\frac{1}{2}gt^2 = -5t^2$)

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ii) At $t = 3$

$$\underline{r} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + 3\begin{pmatrix} 30 \\ 40 \end{pmatrix} - 9\begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

$$\underline{r} = \begin{pmatrix} 90 \\ 80 \end{pmatrix}$$

At $t = 5$

$$\underline{r} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + 5\begin{pmatrix} 30 \\ 40 \end{pmatrix} - 25\begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

$$\underline{r} = \begin{pmatrix} 150 \\ 80 \end{pmatrix}$$

Displacement from $t = 3$ to $t = 5$
$$= \begin{pmatrix} 60 \\ 0 \end{pmatrix}$$

MEI MECHANICS 1 JANUARY 2012

4 iii)

$$x = 30t \quad ①$$

$$y = 5 + 40t - 5t^2 \quad ②$$

From ① $t = \frac{x}{30}$

sub for t in ②

$$y = 5 + \frac{40x}{30} - 5\frac{x^2}{900}$$

$$y = 5 + \frac{4x}{3} - \frac{x^2}{180}$$

5)

$$\underline{p} = 8\underline{i} + \underline{j}$$

$$\underline{q} = 4\underline{i} - 7\underline{j}$$

i)

$$|\underline{p}| = \sqrt{8^2+1^2} = \sqrt{65}$$

$$|\underline{q}| = \sqrt{4^2+(-7)^2} = \sqrt{16+49} = \sqrt{65}$$

$$\therefore |\underline{p}| = |\underline{q}|$$

ii)

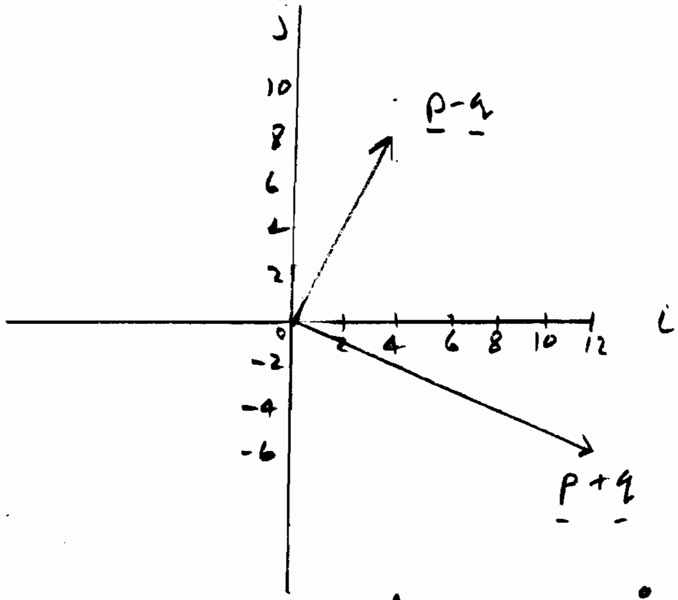
$$\underline{p}+\underline{q} = 8\underline{i}+\underline{j}+4\underline{i}-7\underline{j} = 12\underline{i}-6\underline{j} = 6(2\underline{i}-\underline{j})$$

$\therefore$ $\underline{p}+\underline{q}$ is parallel to $2\underline{i}-\underline{j}$

iii)

$$\underline{p}+\underline{q} = 12\underline{i}-6\underline{j}$$

$$\underline{p}-\underline{q} = (8\underline{i}+\underline{j})-(4\underline{i}-7\underline{j}) = 4\underline{i}+8\underline{j}$$



Angle is 90°

Section B

6) i)

$u = 40\,ms^{-1}$

800 kg

$s = 125\,m$

$$v^2 = u^2 + 2as$$

$$0 = 40^2 + 2 \times a \times 125$$

$$0 = 1600 + 250a$$

$$-1600 = 250a$$

$$-6.4 = a$$

$F = ma$ $\quad F = 800 \times (-6.4)$

$$F = -5120\,N$$

Resistive force of 5120N

MEI MECHANICS 1 JANUARY 2012

6ii)

$$v = u + at$$

$$0 = 40 - 6.4t$$

$$6.4t = 40$$

$$t = \frac{40}{6.4} = 6.25$$

Rest after 6.25 s

6iii) Takes 125m to stop after hitting brakes

Since stops within 155m Robin travels less than 30m at $40 ms^{-1}$ before hitting brakes

$$\therefore \text{reaction time} < \frac{30}{40}$$

$$= 0.75 s$$

6iv) In reaction time car travels

$$0.675 \times 20 = 13.5m$$

Then $v^2 = u^2 + 2as$

$$0 = 20^2 - 2 \times 6.4 \times s$$

$$12.8s = 400$$

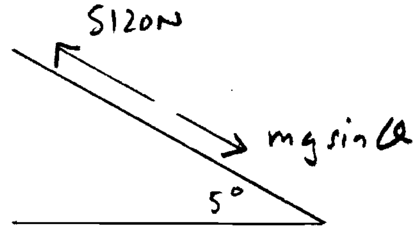
$$s = \frac{400}{12.8} = 31.25m$$

Total stopping distance

$$= 13.5 + 31.25$$

$$= 44.75 m$$

6v)



Resistive force now

$$5120 - 800 \times 9.8 \times \sin 5^\circ$$

$$= 4437N$$

$$a = -\frac{4437}{800} \qquad \left(\frac{F}{m}\right)$$

$$v^2 = u^2 + 2as$$

$$0 = 400 - 2 \times \frac{4437}{800} s$$

$$\frac{4437s}{400} = 400$$

$$s = \frac{400}{\frac{4437}{400}} = 36.06m$$

Reaction distance 13.5m

so stopping distance

$$36.06 + 13.5 = 49.56m$$

6vi)

$$\frac{49.56}{44.75} = 1.107$$

Increased by 10.7%

$\approx$ 11%

MEI MECHANICS 1 JANUARY 2012

7) i) Vertically $y = ut + \frac{1}{2}at^2$

$y = 40\sin\alpha \times t - \frac{1}{2}gt^2$

Lands when $y = 0$

$0 = 40t\sin\alpha - \frac{1}{2}gt^2$

$0 = t\left(40\sin\alpha - \frac{gt}{2}\right)$

$\Rightarrow t = 0$ start

or $40\sin\alpha - \frac{gt}{2} = 0$

$80\sin\alpha - gt = 0$

$80\sin\alpha = gt$

$\frac{80\sin\alpha}{g} = t$

$\therefore T = \frac{80\sin\alpha}{g}$

Horizontally $x = 40\cos\alpha \times t$

$R = 40\cos\alpha \times \frac{80\sin\alpha}{g}$

$R = \frac{3200\sin\alpha\cos\alpha}{g}$

7ii) $\alpha = 30°$ $u = 40\text{ms}^{-1}$

$T = \frac{80\sin 30}{9.8} = 4.08\text{ s}$

$R = \frac{3200\sin 30°\cos 30°}{9.8} = 141.4\text{m}$

$\alpha = 45°$ $T = \frac{80\sin 45}{9.8}$

$T = 5.77\text{ s}$

$R = \frac{3200\sin 45°\cos 45}{9.8}$ 163.3m

iii) Standard model inaccurate predicts 141.4m when actual is 125m which is 13% too much.

iv) $x = u_x t + \frac{1}{2}at^2$

$x = 40\cos 30 \times t - \frac{1}{2}\times 2t^2$

$x = 40t\cos 30 - t^2$

When $t = 4.08$

$x = 40 \times 4.08\cos 30° - 4.08^2$

$x = 124.69\text{ m} \approx 125\text{m}$

v) New $T = 5.77\text{s}$ (from part ii)

$x = 40t\cos 45 - t^2$

when $t = 5.77$

$x = 40 \times 5.77\cos 45° - 5.77^2$

$x = 129.9\text{m}$ which is within 4% of actual

7v) (cont) However, it is not as accurate for 45° as it was for 30°

7vi) Allow for a resistive force in vertical direction