OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **4766**

Statistics 1

Advanced Subsidiary General Certificate of Education

MEI STATISTICS **G241**

Statistics 1 (Z1)

Thursday **12 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
- 8 page answer booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

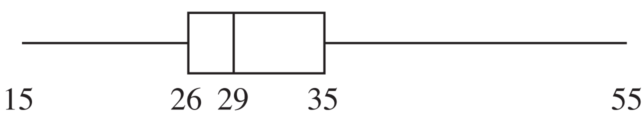
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 6 printed pages and 2 blank pages.

Section A (36 marks)

1 The times taken, in minutes, by 80 people to complete a crossword puzzle are summarised by the box and whisker plot below.



(i) Write down the range and the interquartile range of the times. [2]

(ii) Determine whether any of the times can be regarded as outliers. [3]

(iii) Describe the shape of the distribution of the times. [1]

2 Four letters are taken out of their envelopes for signing. Unfortunately they are replaced randomly, one in each envelope.

The probability distribution for the number of letters, X, which are now in the correct envelope is given in the following table.

r	0	1	2	3	4
$\mathrm{P}(X = r)$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	0	$\frac{1}{24}$

(i) Explain why the case $X = 3$ is impossible. [1]

(ii) Explain why $\mathrm{P}(X = 4) = \frac{1}{24}$. [2]

(iii) Calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [5]

3 Over a long period of time, 20% of all bowls made by a particular manufacturer are imperfect and cannot be sold.

(i) Find the probability that fewer than 4 bowls from a random sample of 10 made by the manufacturer are imperfect. [2]

The manufacturer introduces a new process for producing bowls. To test whether there has been an improvement, each of a random sample of 20 bowls made by the new process is examined. From this sample, 2 bowls are found to be imperfect.

(ii) Show that this does not provide evidence, at the 5% level of significance, of a reduction in the proportion of imperfect bowls. You should show your hypotheses and calculations clearly. [6]

4 A company sells sugar in bags which are labelled as containing 450 grams.

Although the mean weight of sugar in a bag is more than 450 grams, there is concern that too many bags are underweight. The company can adjust the mean or the standard deviation of the weight of sugar in a bag.

(i) State two adjustments the company could make. [2]

The weights, x grams, of a random sample of 25 bags are now recorded.

(ii) Given that $\sum x = 11\,409$ and $\sum x^2 = 5206937$, calculate the sample mean and sample standard deviation of these weights. [3]

5 A school athletics team has 10 members. The table shows which competitions each of the members can take part in.

		Competiton				
		100 m	200 m	110 m hurdles	400 m	Long jump
Athlete	Abel	✓	✓			✓
	Bernoulli		✓		✓	
	Cauchy	✓		✓		✓
	Descartes	✓	✓			
	Einstein		✓		✓	
	Fermat	✓		✓		
	Galois				✓	✓
	Hardy	✓	✓			✓
	Iwasawa		✓		✓	
	Jacobi			✓		

An athlete is selected at random. Events A, B, C, D are defined as follows.

A: the athlete can take part in exactly 2 competitions.
B: the athlete can take part in the 200 m.
C: the athlete can take part in the 110 m hurdles.
D: the athlete can take part in the long jump.

(i) Write down the value of $\mathrm{P}(A \cap B)$. [1]

(ii) Write down the value of $\mathrm{P}(C \cup D)$. [1]

(iii) Which two of the four events A, B, C, D are mutually exclusive? [1]

(iv) Show that events B and D are not independent. [2]

 [Turn over

6 A band has a repertoire of 12 songs suitable for a live performance. From these songs, a selection of 7 has to be made.

(i) Calculate the number of different selections that can be made. [2]

(ii) Once the 7 songs have been selected, they have to be arranged in playing order. In how many ways can this be done? [2]

Section B (36 marks)

7 At East Cornwall College, the mean GCSE score of each student is calculated. This is done by allocating a number of points to each GCSE grade in the following way.

Grade	A*	A	B	C	D	E	F	G	U
Points	8	7	6	5	4	3	2	1	0

(i) Calculate the mean GCSE score, X, of a student who has the following GCSE grades:

A*, A*, A, A, A, B, B, B, B, C, D. [2]

60 students study AS Mathematics at the college. The mean GCSE scores of these students are summarised in the table below.

Mean GCSE score	Number of students
$4.5 \leqslant X < 5.5$	8
$5.5 \leqslant X < 6.0$	14
$6.0 \leqslant X < 6.5$	19
$6.5 \leqslant X < 7.0$	13
$7.0 \leqslant X \leqslant 8.0$	6

(ii) Draw a histogram to illustrate this information. [3]

(iii) Calculate estimates of the sample mean and the sample standard deviation. [5]

The scoring system for AS grades is shown in the table below.

AS Grade	A	B	C	D	E	U
Score	60	50	40	30	20	0

The Mathematics department at the college predicts each student's AS score, Y, using the formula $Y = 13X - 46$, where X is the student's average GCSE score.

(iv) What AS grade would the department predict for a student with an average GCSE score of 7.4? [2]

(v) What do you think the prediction should be for a student with an average GCSE score of 5.5? Give a reason for your answer. [3]

(vi) Using your answers to part **(iii)**, estimate the sample mean and sample standard deviation of the predicted AS scores of the 60 students in the department. [3]

8 Jane buys 5 jam doughnuts, 4 cream doughnuts and 3 plain doughnuts.

On arrival home, each of her three children eats one of the twelve doughnuts. The different kinds of doughnut are indistinguishable by sight and so selection of doughnuts is random.

Calculate the probabilities of the following events.

(i) All 3 doughnuts eaten contain jam. [3]

(ii) All 3 doughnuts are of the same kind. [3]

(iii) The 3 doughnuts are all of a different kind. [3]

(iv) The 3 doughnuts contain jam, given that they are all of the same kind. [3]

On 5 successive Saturdays, Jane buys the same combination of 12 doughnuts and her three children eat one each. Find the probability that all 3 doughnuts eaten contain jam on

(v) exactly 2 Saturdays out of the 5, [3]

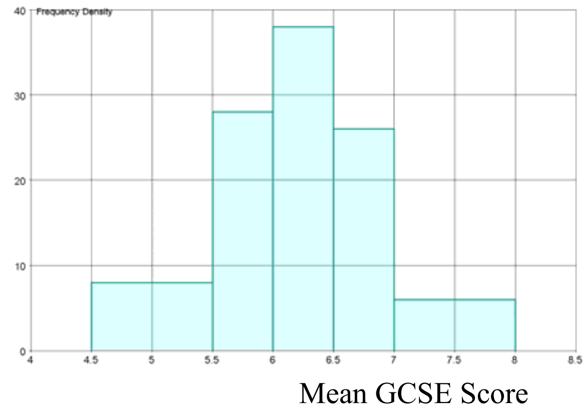
(vi) at least 1 Saturday out of the 5. [3]

Mark Scheme 4766
January 2006

Q 1 **(i)**	The range = 55 – 15 = 40 The interquartile range = 35 – 26 = 9	B1 CAO B1 CAO	**2**
(ii)	35 + 1.5 x 9 = 48.5 26 – 1.5 x 9 = 12.5 Any value > 48.5 is an outlier (so 55 will be an outlier),	M1 for 48.5 oe M1 for 12.5 oe A1 (FT their IQR in (i))	**3**
(iii)	One valid comment such as eg: Positively skewed Middle 50% of data is closely bunched	E1	**1**
		TOTAL	**6**
2 **(i)**	Impossible because if 3 letters are correct, the fourth must be also.	E1	**1**
(ii)	There is only one way to place letters correctly. There are 4! = 24 ways to arrange 4 letters. OR: $\frac{1}{4}$ x $\frac{1}{3}$ x $\frac{1}{2}$ NOTE: **ANSWER GIVEN**	E1 E1 B1 for $\frac{1}{4}$ x $\frac{1}{3}$ B1 for x $\frac{1}{2}$	**2**
(iii)	$E(X) = 1 \times \frac{1}{3} + 2 \times \frac{1}{4} + 4 \times \frac{1}{24} = 1$ $E(X^2) = 1 \times \frac{1}{3} + 4 \times \frac{1}{4} + 16 \times \frac{1}{24} = 2$ So $\mathrm{Var}(X) = 2 - 1^2$ $= 1$	M1 For $\sum xp$ (at least 2 non-zero terms correct) A1 CAO M1 for $\sum \boldsymbol{x}^2\boldsymbol{p}$ (at least 2 non-zero terms correct) M1*dep* for – their $E(X)^2$ A1 FT their E(X) provided $\mathrm{Var}(X) > 0$	**5**
		TOTAL	**8**

3 **(i)**	$X \sim B(10,0.2)$ $P(X < 4) = P(X \le 3) = 0.8791$ OR attempt to sum $P(X = 0,1,2,3)$ using $X \sim B(10,0.2)$ can score M1, A1	M1 for $X \le 3$ A1	**2**
(ii)	Let p = the probability that a bowl is imperfect $H_0 : p = 0.2 \quad H_1 : p < 0.2$ $X \sim B(20,0.2)$ $P(X \le 3) = 0.2061$ $0.2061 > 5\%$ Cannot reject H_0 and so insufficient evidence to claim a reduction. OR using critical region method: CR is {0} B1, 2 not in CR M1, A1 as above	B1 Definition of p B1, B1 B1 for 0.2061 seen M1 for this comparison A1 *dep* for comment <u>in context</u>	**3** **3**
		TOTAL	**8**
4 **(i)**	The company could increase the mean weight. The company could decrease the standard deviation.	B1 CAO B1	**2**
(ii)	Sample mean = 11409/25 = 456.36 $S_{xx} = 5206937 - \frac{11409^2}{25} = 325.76$ Sample s.d = $\sqrt{\frac{325.76}{24}}$ =3.68	B1 M1 for S_{xx} A1	**3**
		TOTAL	**5**
5 **(i)**	P(A ∩ B) = 0.4	B1 CAO	**1**
(ii)	P(C U D) = 0.6	B1 CAO	**1**
(iii)	Events B and C are mutually exclusive.	B1 CAO	**1**
(iv)	P(B) = 0.6, P(D) = 0.4 and P(B ∩ D) = 0.2 $0.6 \times 0.4 \ne 0.2$ (so B and D not independent)	B1 for P(B ∩ D) = 0.2 soi E1	**2**
		TOTAL	**5**
6 **(i)**	Number of selections = $\binom{12}{7}$ = 792	M1 for $\binom{12}{7}$ A1 CAO	**2**
(ii)	Number of arrangements = 7! = 5040	M1 for 7!, A1 CAO	**2**
		TOTAL	**4**

<table>
<tr><td>7
(i)</td><td>Mean score = (2x8 + 3x7 + 4x6 + 5 + 4)/11 =

6.36</td><td>M1 for $\sum fx/11$
A1 CAO</td><td>2</td></tr>
<tr><td>(ii)</td><td>
</td><td>G1 Linear sensible scales

G1 fds of 8, 28, 38, 26, 6 or 4k, 14k, 19k, 13k, 3k for sensible values of k either on script or on graph.

G1 (dep on reasonable attempt at fd) Appropriate label for vertical scale eg ‘Frequency density’, ‘frequency per ½ unit’, ‘students per mean GCSE score’. (allow Key)</td><td>3</td></tr>
<tr><td>(iii)</td><td><table>
<tr><th>Mid point, x</th><th>f</th><th>fx</th><th>fx²</th></tr>
<tr><td>5</td><td>8</td><td>40</td><td>200</td></tr>
<tr><td>5.75</td><td>14</td><td>80.5</td><td>462.875</td></tr>
<tr><td>6.25</td><td>19</td><td>118.75</td><td>742.1875</td></tr>
<tr><td>6.75</td><td>13</td><td>87.75</td><td>592.3125</td></tr>
<tr><td>7.5</td><td>6</td><td>45</td><td>337.5</td></tr>
<tr><td></td><td>60</td><td>372</td><td>2334.875</td></tr>
</table>
Sample mean = 372/60 = 6.2

$S_{xx} = 2334.875 - \frac{372^2}{60} = 28.475$

Sample s.d = $\sqrt{\frac{28.475}{59}} = 0.695$</td><td>B1 mid points

B1FT $\sum fx$ and $\sum fx^2$

B1 CAO

M1 for their S_{xx}

A1 CAO</td><td>5</td></tr>
<tr><td>(iv)</td><td>Prediction of score = 13 x 7.4 – 46 = 50.2
So predicted AS grade would be B</td><td>M1 For 13 x 7.4 – 46
A1 dep on 50.2 (or 50) seen</td><td>2</td></tr>
<tr><td>(v)</td><td>Prediction of score = 13 x 5.5 – 46 = 25.5

So predicted grade would be D/E
(allow D or E)
Because score roughly halfway from 20 to 30,
OR (for D) closer to D than E
OR (for E) past E but not up to D boundary</td><td>M1 For 13 x 5.5 – 46

A1 dep on 25.5 (or 26 or 25) seen
E1 For explanation of conversion – logical statement/argument that supports their choice.</td><td>3</td></tr>
<tr><td>(vi)</td><td>Mean = 13 x 6.2 – 46 = 34.6
Standard deviation = 13 x 0.695 = 9.035</td><td>B1 FT their 6.2
M1 for 13 x their 0.695
A1 FT</td><td>3</td></tr>
<tr><td></td><td></td><td>TOTAL</td><td>18</td></tr>
</table>

8 (i)	P(all jam) $= \frac{5}{12}\times\frac{4}{11}\times\frac{3}{10}$ $= \frac{1}{22} = 0.04545$	M1 5 x 4 x 3 or $\binom{5}{3}$ in numerator M1 12 x 11 x 10 or $\binom{12}{3}$ in denominator A1 CAO	**3**
(ii)	P(all same) $= \frac{5}{12}\times\frac{4}{11}\times\frac{3}{10} + \frac{4}{12}\times\frac{3}{11}\times\frac{2}{10} + \frac{3}{12}\times\frac{2}{11}\times\frac{1}{10}$ $= \frac{1}{22}+\frac{1}{55}+\frac{1}{220} = \frac{3}{44} = 0.06818$	M1 Sum of 3 reasonable triples or combinations M1 Triples or combinations correct A1 CAO	**3**
(iii)	P(all different) $= 6\times\frac{5}{12}\times\frac{4}{11}\times\frac{3}{10}$ $= \frac{3}{11} = 0.2727$	M1 5,4,3 M1 $6\times$ three fractions or $\binom{12}{3}$ denom. A1 CAO	**3**
(iv)	P(all jam given all same) = $\frac{1}{22}\Big/\frac{3}{44} = \frac{2}{3}$	M1 Their (i) in numerator M1 Their (ii) in denominator A1 CAO	**3**
(v)	P(all jam exactly twice) $= \binom{5}{2}\times\left(\frac{1}{22}\right)^2\times\left(\frac{21}{22}\right)^3 = 0.01797$	M1 for $\binom{5}{2}$x … M1 for their $p^2 q^3$ A1 CAO	**3**
(vi)	P(all jam at least once) $= 1-\left(\frac{21}{22}\right)^5 = 0.2075$	M1 for their q^5 M1 indep for 1 – 5^{th} power A1 CAO	**3**
		TOTAL	**18**

4766: Statistics 1

General Comments

Overall many candidates were able to make a good attempt at all of the questions, with the exception of question 5 which many found difficult. It is pleasing to note that there were very few scripts where candidates seemed to have no idea of how to tackle almost anything in the paper. The new work on expectation and variance of a discrete random variable was well answered. Centres should remind their candidates that in any question involving the binomial distribution that the definition of p should be clearly stated in the solution. Many candidates were familiar with the new formula for the sample standard deviation with the divisor of $(n – 1)$ but the examiners did equally see many candidates using a divisor of n, which in the new specification is defined as the rmsd (root mean squared deviation).

The two longer questions in section B attracted good responses with question 7 proving more popular than question 8. There was some evidence, judging by the incomplete attempts at question 8, that candidates had not divided their time sensibly across the paper. This was often linked to candidates using time consuming methods, such as calculation of multiple binomial probabilities when the answer could be found in tables, or recalculating mean and standard deviation from scratch, when coding could be used.

Candidates should also be reminded that they should show sufficient working in the calculation of the mean and standard deviation. Many preferred to state answers only (often ruthlessly rounded) and it was then impossible to award the appropriate method marks. Another source of lost marks was through premature approximation of answers which were then used in further calculations.

Comments on Individual Questions

Section A

1) **Times to complete a crossword puzzle: range, IQR, outliers and description of the distribution**
This question was generally well answered with many candidates gaining high marks and almost all gaining both marks for part (i). The formula for identifying outliers in terms of 1.5 x IQR from the relevant quartiles was well known but there was a minority of candidates who believed it was referenced from the median rather than the quartiles, or used a multiplier of 2 instead of 1.5. Part (iii) was usually answered correctly but a number of candidates used the phrase 'a positive distribution' rather than positively skewed. Negative instead of positive skew was sometimes seen.

2) **Letters in envelopes: Probability distribution. Calculation of E(*X*) and Var(*X*)**
The explanations in parts (i) and (ii) were usually convincing but many candidates gave a simplistic response to both parts by trying to justify that the sum of the probabilities was unity. Whilst this was true, it did not answer the questions posed. The calculation of $E(X)$ and $Var(X)$ was usually well answered with only the occasional candidate using $\sum xp^2$ or $\sum(xp)^2$ instead of the correct $\sum x^2p$. Other occasional errors included division of the correct value of $E(X)$ by 5, and failure to subtract $(E(X))^2$ from $E(X^2)$.

3) (i) **The binomial distribution: Imperfect bowls. Hypothesis test on *p***

This was generally well answered, although a number of candidates wasted time calculating probabilities, rather than using the binomial tables. Those that did use summative probabilities often floundered by omitting $P(X = 0)$ or were found wanting through premature approximation of their answers. Too many found $P(X \leq 4)$, $P(X = 4)$ or $1 - P(X \leq 3)$ instead of the required $P(X \leq 3)$.

(ii) In the hypothesis test, although many candidates gave correct hypotheses in terms of p, few defined p explicitly in words. Centres should advise candidates that such a definition does attract credit. It was notable that from any given centre it was usually the case that either almost all candidates defined p or no candidates did so. The hypotheses themselves were usually correctly given but a number of candidates still continue to lose marks through poor notation. Candidates should be aware that $H_0 = 0.2$ is **not** an acceptable notation, nor is $H_0 : P(X=0.2)$. The standard notation is $H_0: p = 0.2$. As in previous sessions, many candidates used point probabilities, which effectively prevents any further credit being gained. Those who were successful in comparing the tail probability of 0.2061 with 0.05 often lost the final mark by not putting their conclusion in context. To simply state 'Accept H_0' on its own is not sufficient to gain credit here. A conclusion along the lines of 'There is insufficient evidence to claim that there has been a reduction' is needed to gain the mark.

An argument based on critical regions is of course perfectly acceptable, but candidates preferring to use such arguments need to be very precise. To simply state that the critical region is $\{0\}$ without a probability justification is insufficient.

4) (i) **Sugar bags: Adjustments the company could make. Mean and sample standard deviation**

Comments often made no mention of the mean and standard deviation. Vacuous remarks such as 'put more sugar in', 'use bigger bags' or 'install more accurate machinery' etc were commonplace. Those that did use the mean and standard deviation were equally non committal with statements along the lines of 'adjust the mean', 'adjust the standard deviation'. Again the reasoning needs to be clear. '**Increase** the mean' and '**decrease** the standard deviation' were the answers required.

(ii) The calculation of the sample mean and sample standard deviation were usually correct but a number of candidates calculated root mean squared deviation with a divisor of n. The specification explicitly defines standard deviation with a divisor of $n - 1$ and candidates should be aware of this. A few candidates found variance correctly but then forgot to take the square root. Some candidates used the old formula $\sqrt{(\sum x^2/n - ()^2)}$ and whilst such candidates were on this occasion given credit, candidates should be aware that this approach will only in future attract any credit at all if it leads to a fully correct answer. In the calculation of any of the relevant measures of dispersion, candidates are **strongly recommended** to first calculate the sum of squares S_{xx}. The data in this question were such that premature approximation made a very large difference to the final answer for standard deviation and it is pleasing that such approximation was relatively uncommon.

5) **Athletes team: Probability rules. Mutually exclusive events. Test for independent events**

Candidates found this question difficult. Most were unaware that the answers could easily be obtained by counting the relevant ticks in the table. Instead many resorted to inappropriate use of formulae. Many assumed the events to be independent or mutually exclusive when they were not.

(i) Many candidates simply found P(A) × P(B), assuming independence.

(ii) Many candidates found $P(C \cup D) = \frac{3}{10} + \frac{4}{10}$, often then subtracting $\frac{3}{10} \times \frac{4}{10}$ instead of the correct form $P(C \cup D) = \frac{3}{10} + \frac{4}{10} - \frac{1}{10}$.

(iii) Many candidates gained credit here for identifying the mutually exclusive events.

(iv) A number of candidates correctly used a test for independence, most popularly $P(A \cap B) = P(A) \times P(B)$, or less often a test based on conditional probability. However there were many incorrect qualitative arguments seen, often noting that some athletes took part in both events B and D.

6) **Selection of songs at a performance**

This was the best answered question on the paper with a sizeable majority scoring full marks. Part (i) was invariably correct but a variety of incorrect responses was seen in part (ii) with $^{12}P_7$, 7^7 and 7^2 instead of 7! being the most popular of these.

7) **GCSE and A level grades: Mean. Histogram. Sample mean and Sample standard deviation. Linear coding of data**

There were some excellent responses to this question.

(i) Most candidates scored both marks.

(ii) Most candidates were aware of how to construct a histogram correctly either via a frequency density or frequency per standard interval approach. Those who chose the latter route unfortunately often simply labelled the vertical axis as ‘frequency density’ rather than eg ‘frequency per 0.5 GCSE points’ . There were equally many erroneous constructions seen by the examiners ranging from a simple frequency plot to frequency divided by mid-points plot and even frequency × class widths plots.

(iii) Many candidates used the mid-points correctly to calculate the sample mean and sample standard deviation, but frequently rounding errors led to inaccuracies. Candidates who insist on giving answers only (with no supportive working) to this type of question must do so with great care. Often the examiners saw incorrect answers followed by the legend ‘calc used’. Unfortunately no marks could be awarded for such a response. As mentioned before, there is still a minority of candidates who mistakenly calculated the rmsd instead of the sample standard deviation. Likewise some candidates did not deal correctly with the frequencies, calculating $\sum xf^2$ or $\sum(xf)^2$ instead of the correct $\sum x^2 f$.

(iv) Many candidates achieved the answer of 50.2 but some failed to follow it up with a declaration that it was equivalent to ‘grade B’. A small number did the reverse, offering an answer of grade B without the required supporting evidence.

(v) Curiously 5.5 was often not substituted into the formula. Comments were often based on personal feelings such as ‘predict a higher grade to encourage them’ rather than interpreting the value of 25.5 in the context of the table given in the question.

(vi) The coded mean was usually correct but many applied the '-46' to the sample standard deviation or simply said 'it would not change'. A substantial number of candidates wasted an inordinate amount of time by recalculating a 'new' set of data using the given formula, often making errors along the way.

8) **Probability methods applied to selecting doughnuts. Conditional probability. Binomial distribution calculations.**

Many candidates scored well in this question with some gaining full or near to full marks.

(i)(ii) Many scored full marks in both parts. However a minority did the whole question based on 'with replacement' for which some allowance was made. Another error occasionally seen was an attempt to use a binomial distribution with $n = 12$. The use of fractions rather than decimals is strongly advisable for a question such as this. Candidates using decimals usually had rounding errors that got worse as the question went on.

(iii) Multiplication by 3 rather than 3! was common and equally often no multiplier at all was seen. Another common error here was to believe that the answer was 1 – answer (ii).

(iv) The conditional probability was usually answered correctly, again with a fairly generous follow through based on the earlier answers. However some candidates applied the conditional probability formula and then wrote down a denominator of $P(A) \times P(B)$ which is of course only correct if A and B are independent. This is extremely unlikely to be the case in a conditional probability question.

(v)(vi) Both parts proved difficult for many candidates. Although often candidates realised that a binomial distribution was appropriate many of them used the wrong parameters, often using $p = {}^{5}/_{12}$, or in some cases omitted the combination factor. Others did not recognise that they should apply a binomial model. Those who used correct methods often had rounding errors at this point. The attempts at 'at least one' in part (vi) were generally successful but the examiners did occasionally see $1 - \{P(X=0) + P(X=1)\}$ or even just the calculation of $P(X=1)$ appearing in the work.