

ADVANCED SUBSIDIARY GCE MATHEMATICS (MEI)

4766

Statistics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Monday 19 January 2009 Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.
- This document consists of 8 pages. Any blank pages are indicated.

Section A (36 marks)

A supermarket chain buys a batch of 10000 scratchcard draw tickets for sale in its stores. 50 of these 1 tickets have a £10 prize, 20 of them have a £100 prize, one of them has a £5000 prize and all of the rest have no prize. This information is summarised in the frequency table below.

Prize money	£0	£10	£100	£5000
Frequency	9929	50	20	1

- (i) Find the mean and standard deviation of the prize money per ticket. [4]
- (ii) I buy two of these tickets at random. Find the probability that I win either two $\pounds 10$ prizes or two £100 prizes. [3]
- 2 Thomas has six tiles, each with a different letter of his name on it.
 - (i) Thomas arranges these letters in a random order. Find the probability that he arranges them in the correct order to spell his name. [2]
 - (ii) On another occasion, Thomas picks three of the six letters at random. Find the probability that he picks the letters T, O and M (in any order). [3]
- 3 A zoologist is studying the feeding behaviour of a group of 4 gorillas. The random variable Xrepresents the number of gorillas that are feeding at a randomly chosen moment. The probability distribution of X is shown in the table below.

r	0	1	2	3	4
$\mathbf{P}(X=r)$	р	0.1	0.05	0.05	0.25

(i)	Find the value of <i>p</i> .	[1]

- (ii) Find the expectation and variance of X.
- (iii) The zoologist observes the gorillas on two further occasions. Find the probability that there are at least two gorillas feeding on both occasions. [2]
- A pottery manufacturer makes teapots in batches of 50. On average 3% of teapots are faulty. 4
 - (i) Find the probability that in a batch of 50 there is

(A)	exactly one faulty teapot,	[3]
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- (B) more than one faulty teapot. [3]
- (ii) The manufacturer produces 240 batches of 50 teapots during one month. Find the expected number of batches which contain exactly one faulty teapot. [2]

[5]

- 5 Each day Anna drives to work.
 - *R* is the event that it is raining.
 - *L* is the event that Anna arrives at work late.

You are given that P(R) = 0.36, P(L) = 0.25 and $P(R \cap L) = 0.2$.

- (i) Determine whether the events *R* and *L* are independent. [2]
- (ii) Draw a Venn diagram showing the events *R* and *L*. Fill in the probability corresponding to each of the four regions of your diagram. [3]

[3]

(iii) Find P(L|R). State what this probability represents.

[Question 6 is printed overleaf.]

Section B (36 marks)

6 The temperature of a supermarket fridge is regularly checked to ensure that it is working correctly. Over a period of three months the temperature (measured in degrees Celsius) is checked 600 times. These temperatures are displayed in the cumulative frequency diagram below.



- (i) Use the diagram to estimate the median and interquartile range of the data. [3]
- (ii) Use your answers to part (i) to show that there are very few, if any, outliers in the sample. [4]
- (iii) Suppose that an outlier is identified in these data. Discuss whether it should be excluded from any further analysis. [2]
- (iv) Copy and complete the frequency table below for these data.

Temperature (<i>t</i> degrees Celsius)	$3.0 \leq t \leq 3.4$	$3.4 < t \le 3.8$	$3.8 < t \le 4.2$	$4.2 < t \le 4.6$	$4.6 < t \le 5.0$
Frequency			243	157	

- (v) Use your table to calculate an estimate of the mean.
- (vi) The standard deviation of the temperatures in degrees Celsius is 0.379. The temperatures are converted from degrees Celsius into degrees Fahrenheit using the formula F = 1.8C + 32. Hence estimate the mean and find the standard deviation of the temperatures in degrees Fahrenheit. [3]

[2]

[3]

- 7 An online shopping company takes orders through its website. On average 80% of orders from the website are delivered within 24 hours. The quality controller selects 10 orders at random to check when they are delivered.
 - (i) Find the probability that
 - (A) exactly 8 of these orders are delivered within 24 hours, [3]
 - (B) at least 8 of these orders are delivered within 24 hours. [2]

The company changes its delivery method. The quality controller suspects that the changes will mean that fewer than 80% of orders will be delivered within 24 hours. A random sample of 18 orders is checked and it is found that 12 of them arrive within 24 hours.

- (ii) Write down suitable hypotheses and carry out a test at the 5% significance level to determine whether there is any evidence to support the quality controller's suspicion. [7]
- (iii) A statistician argues that it is possible that the new method could result in either better or worse delivery times. Therefore it would be better to carry out a 2-tail test at the 5% significance level. State the alternative hypothesis for this test. Assuming that the sample size is still 18, find the critical region for this test, showing all of your calculations. [7]

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4766 Statistics 1

Section A

Q1 (i)	(With $\sum fx = 7500$ and $\sum f = 10000$ then arriving at the		
	mean)(i)£0.75 scores (B1, B1)(ii)75p scores (B1, B1)(iii)0.75p scores (B1, B0) (incorrect units)(iv)£75 scores (B1, B0) (incorrect units)After B0, B0then sight of $\frac{7500}{10000}$ scores SC1. SC1or an answerin the range £0.74 - £0.76 or 74p - 76p (both inclusive) scoresSC1 (units essential to gain this mark)	B1 for numerical mean (0.75 or 75 seen) B1dep for correct units attached	
	 Standard Deviation: (CARE NEEDED here with close proximity of answers) 50.2(0) using divisor 9999 scores B2 (50.20148921) 50.198 (= 50.2) using divisor 10000 scores B1(<i>rmsd</i>) If divisor is <u>not</u> shown (or calc used) and only an answer of 50.2 (i.e. <u>not</u> coming from 50.198) is seen then award B2 on b.o.d. (default) 	B2 correct s.d. (B1) correct rmsd (B2) default	
	After B0 scored then an attempt at S_{xx} as evident by either $S_{xx} = (5000 + 200000 + 25000000) - \frac{7500^2}{10000}$ (= 25199375) or $S_{xx} = (5000 + 200000 + 25000000) - 10000(0.75)^2$ scores (M1) or M1ft 'their 7500 ² ' or 'their 0.75 ² ' NB The <u>structure</u> must be correct in both above cases with a max of <u>1 slip only after applying the f.t.</u>	$\sum fx^2 = 25,205,000$ Beware $\sum x^2 = 25,010,100$ After B0 scored then (M1) or M1f.t. for attempt at S_{xx} NB full marks for correct results from recommended method which is use of calculator functions	4

$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	(ii)	P(Two £10 or two £100)		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$		$-50 \times 49 \times 20 \times 19$	M1 for either correct	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		$-\frac{1}{10000} \times \frac{1}{9999} + \frac{1}{10000} \times \frac{1}{9999}$	product seen	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		= 0.0000245 + 0.0000038 = (0.00002450245 + 0.00000380038)	M1 sum of both correct	
After M0, M0 then $\frac{50}{10000} \times \frac{50}{10000} \times \frac{20}{10000} \times \frac{20}{10000}$ o.e. Al CAO (as opposite with no rounding) 3 Scores SC1 (ignore final answer but SC1 may be implied by sight of 2.9 × 10 ⁻⁵ o.e.) Similarly, $\frac{50}{10000} \times \frac{49}{10000} \times \frac{49}{10000} \times \frac{19}{10000}$ scores SC1 SC1 case #1) (SC1 case #2) CARE mawer is also 2.83 \times 10^{-5} 7 Image: Solid score scor		= 0.000028(3) o.e. = (0.00002830283)	(ignore any multipliers)	
Intervision with 10000 $^{-1}$ 10000 $^{-1}$ 10000 $^{-1}$ 10000 $^{-1}$ with no rounding) with no rounding) 3 Scores SC1 (ignore final answer but SC1 may be implied by sight of 2.9 × 10 $^{-5}$ o.c.) Similarly, $\frac{50}{10000} \times \frac{49}{10000} \times \frac{49}{10000} \times \frac{19}{10000}$ scores SC1 (SC1 case #1) (SC1 case #2) CARE answer is also 2.83 × 10 $^{-5}$ 7 Q2 (i) Either P(all correct) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{720}$ M1 for 6! Or 720 (sice) or product of fractions A1 CAO (accept 0.0014) 2 (ii) Either P(picks T, O, M) = $\frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{20}$ M1 for denominators M1 for or enominators 3 (iii) Either P(picks T, O, M) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! = \frac{1}{20}$ M1 for for $\frac{1}{3}$ or 20 size 3 (iii) Either P(picks T, O, M) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! = \frac{1}{20}$ M1 for $1/\binom{6}{3}$ or 20 size 3 (iii) $p = 0.55$ B1 cao 1 1 (iii) $p = 0.55 + 1 \times 0.1 + 2 \times 0.05 + 3 \times 0.05 + 4 \times 0.25 = 1.35$ M1 for $\Sigma^2 p$ (at least 3 non zero terms correct) A1 CAO(no 'n' or 'n-1' divisors) 1 (iii) $E(X^2) = 0 \times 0.55 + 1 \times 0.1 + 4 \times 0.05 + 9 \times 0.05 + 16 \times 0.25$ M1 for $\Sigma^2 p$ (at least 3 non zero terms correct) A1 CAO(no 'n' or 'n-1' divisors) A1 cao (no 'n' or 'n-1' divisors) $Var($		After M0 M0 then $\frac{50}{50} \times \frac{50}{50} + \frac{20}{50} \times \frac{20}{50}$ o.e.	A1 CAO (as opposite	
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sight of 2.9 × 10 ⁻⁵ o.c.) Similarly, $\frac{50}{10000} \times \frac{49}{10000} + \frac{20}{10000} \times \frac{19}{10000}$ scores SC1 (SC1 case #1) (SC1 case #2) <u>CARE</u> answer is also 2.83 × 10 ⁻⁵ TOTAL 7 Q2 (i) Either P(all correct) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{720}$ or P(all correct) = $\frac{1}{6!} = \frac{1}{720} = 0.00139$ X TOTAL 7 M1 for 6! Or 720 (sicc) or product of fractions A1 CAO (accept 0.0014) 2 (ii) Either P(picks T, O, M) = $\frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{20}$ or P(picks T, O, M) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! = \frac{1}{20}$ or P(picks T, O, M) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! = \frac{1}{20}$ or P(picks T, O, M) = $\frac{1}{6} = \frac{1}{20}$ (iii) E(X) = 0 × 0.55 + 1 × 0.1 + 2 × 0.05 + 3 × 0.05 + 4 × 0.25 = 1.35 E(X ²) = 0 × 0.55 + 1 × 0.1 + 2 × 0.05 + 3 × 0.05 + 4 × 0.25 = 1.35 E(X ²) = 0 × 0.55 + 1 × 0.1 + 4 × 0.05 + 9 × 0.05 + 16 × 0.25 = 0 + 0.1 + 0.2 + 0.45 + 4 = (4.75) Var(X) = 'their' 4.75 - 1.35 ² = 2.9275 awfw (2.9275 - 2.93) (iii) P(At least 2 both times) = (0.05+0.05+0.25) ² = 0.1225 o.e. N1 for (0.05+0.05+0.25) ² O(0.05+0.05+0.25) ² = 0.1225 o.e. (iv) O(0.05+0.05+0.25) ² = 0.1225 o.e.		Scores SC1 (ignore final answer but SC1 may be implied by	$(\mathbf{SC}_{1}, \mathbf{a}, \mathbf{a}$	3
Similarly, $\frac{50}{10000} \times \frac{49}{10000} \times \frac{19}{10000} \times \frac{19}{10000}$ scores SC1 (SC1 case #2) <u>CARE</u> answer is also 2.83 × 10 ⁻³ Q2 (i) Either P(all correct) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{720}$ M1 for 6! Or 720 (sioc) or product of fractions A1 CAO (accept 0.0014) 2 (ii) Either P(picks T, O, M) = $\frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{20}$ M1 for of enominators A1 CAO (accept 0.0014) 2 (iii) Either P(picks T, O, M) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! = \frac{1}{20}$ M1 for denominators or 3! A1 CAO 3 or P(picks T, O, M) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! = \frac{1}{20}$ Or M1 for $\binom{6}{3}$ or 20 since M1 for numerators or 3! A1 CAO 3 (ii) $p = 0.55$ B1 cao 1 (iii) $p = 0.55 + 1 \times 0.1 + 2 \times 0.05 + 3 \times 0.05 + 4 \times 0.25 = 1.35$ M1 for Σrp (at least 3 non zero terms correct) A1 CAO(no 'n' or 'n-1' divisors) 1 $p = 0.55 + 1 \times 0.1 + 2 \times 0.05 + 9 \times 0.05 + 16 \times 0.25$ M1 for Σrp (at least 3 non zero terms correct) A1 CAO(no 'n' or 'n-1' divisors) 1 $F(X^2) = 0 \times 0.55 + 1 \times 0.1 + 4 \times 0.05 + 9 \times 0.05 + 16 \times 0.25$ M1 for Σrp (at least 3 non zero terms correct) A1 CAO(no 'n' or 'n-1' divisors) 1 $F(X^2) = 0 \times 0.55 + 1 \times 0.1 + 4 \times 0.05 + 9 \times 0.05 + 16 \times 0.25$ M1 for $(0.05 + 0.50 + 0.25)^2$ or $(0.35^2 seen A1 + 0.2 + 0.45 + 4)$ 1 1 $F(X^2) = 0 \times 0.55 + $		sight of 2.9×10^{-3} o.e.)	(SC1 case #1)	
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Observe to the form of the second	(i)	Either P(all correct) = $\frac{1}{1} \times \frac{1}{2} = \frac{1}{122}$	M1 for 6! Or 720 (sioc)	
or P(all correct) = $\frac{1}{6!} = \frac{1}{720} = 0.00139$ A1 CAO (accept 0.0014)2(ii)Either P(picks T, O, M) = $\frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{20}$ M1 for denominatorsM1 for numerators or 3!or P(picks T, O, M) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! = \frac{1}{20}$ M1 for numerators or 3!3or P(picks T, O, M) = $\frac{1}{6} = \frac{1}{20}$ Or M1 for $\binom{6}{3}$ or 20 sinceM1 for 1/ $\binom{6}{3}$ 3 $\frac{Q3}{(i)}$ $p = 0.55$ B1 cao1 (ii) $E(X) =$ N1 for $\sum p$ (at least 3)1 $0 \times 0.55 + 1 \times 0.1 + 2 \times 0.05 + 3 \times 0.05 + 4 \times 0.25 = 1.35$ M1 for $\sum p^2$ (at least 3)1 (iii) $E(X) =$ $0 \times 0.55 + 1 \times 0.1 + 4 \times 0.05 + 9 \times 0.05 + 16 \times 0.25$ M1 for $\sum p^2$ (at least 3)1 $Var(X) = "their" 4.75 - 1.35^2 = 2.9275 awfw (2.9275 - 2.93)$ M1 for $\sum p^2$ (at least 3)1 (iii) P(At least 2 both times) = $(0.05+0.05+0.25)^2 = 0.1225$ o.e.M1 for $(0.05+0.05+0.25)^2$ 3 (iii) P(At least 2 both times) = $(0.05+0.05+0.25)^2 = 0.1225$ o.e.M1 for $(0.05+0.05+0.25)^2$ 5	(1)	6 5 4 3 2 1 720	or product of fractions	
(ii)Either P(picks T, O, M) = $\frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{20}$ M1 for denominatorsor P(picks T, O, M) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! = \frac{1}{20}$ M1 for numerators or 3! A1 CAOM1 for numerators or 3! A1 CAOor P(picks T, O, M) = $\frac{1}{6} (\frac{1}{3}) = \frac{1}{20}$ M1 for $1/(\frac{6}{3})$ A1 CAOM1 for $1/(\frac{6}{3})$ A1 CAO3 $\frac{0}{10}$ $\frac{1}{6} (\frac{1}{3}) = \frac{1}{20}$ M1 for $1/(\frac{6}{3})$ A1 CAO $\frac{1}{10}$ $\frac{0}{10}$ $\frac{1}{6} (\frac{1}{3}) = \frac{1}{20}$ $\frac{1}{10}$ $\frac{1}{10} (\frac{1}{3}) = \frac{1}{20}$ $\frac{0}{10}$ $\frac{1}{6} (\frac{1}{3}) = \frac{1}{20}$ $\frac{1}{10}$ $\frac{1}{10} (\frac{1}{3}) = \frac{1}{20}$ $\frac{0}{10}$ $\frac{1}{6} (\frac{1}{3}) = \frac{1}{20}$ $\frac{1}{10} (\frac{1}{3}) = \frac{1}{20}$ $\frac{1}{10} (\frac{1}{3}) = \frac{1}{20}$ $\frac{0}{10}$ $\frac{1}{6} (\frac{1}{3}) = \frac{1}{20}$ $\frac{1}{10} (\frac{1}{3}) = \frac{1}{20}$ $\frac{1}{10} (\frac{1}{10}) = \frac{1}{10} (\frac{1}{3}) = \frac{1}{20}$ $\frac{0}{10}$ $\frac{1}{6} (\frac{1}{3}) = \frac{1}{20}$ $\frac{1}{10} (\frac{1}{3}) = \frac{1}{20}$ $\frac{1}{10} (\frac{1}{10}) = \frac{1}{10} (\frac{1}{3}) = \frac{1}{20} (\frac{1}{3}) = $		or P(all correct) = $\frac{1}{1} = \frac{1}{1} = 0.00139$	1	
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(ii)Either P(picks T, O, M) = $\frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{20}$ or P(picks T, O, M) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! = \frac{1}{20}$ or P(picks T, O, M) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! = \frac{1}{20}$ M1 for denominators M1 for numerators or 3! A1 CAO3(ii) $p = 0.55$ (iii)B1 cao1(iii) $E(X) =$ $0 \times 0.55 + 1 \times 0.1 + 2 \times 0.05 + 3 \times 0.05 + 4 \times 0.25 = 1.35$ $= 0 + 0.1 + 0.2 + 0.45 + 4$ $= (4.75)$ M1 for Σrp (at least 3 non zero terms correct) A1 CAO(no 'n' or 'n-1') divisors)3(iii) $E(X^2) = 0 \times 0.55 + 1 \times 0.1 + 4 \times 0.05 + 9 \times 0.05 + 16 \times 0.25$ $= 0 + 0.1 + 0.2 + 0.45 + 4$ $= (4.75)$ M1 for $\Sigma r^2 p$ (at least 3 non zero terms correct) A1 CAO(no 'n' or 'n-1') divisors)M1 for $\Sigma r^2 p$ (at least 3 non zero terms correct) A1 CAO(no 'n' or 'n-1') divisors)5(iii)P(At least 2 both times) = $(0.05 + 0.05 + 0.25)^2 = 0.1225$ o.e.M1 for $(0.05 + 0.05 + 0.05 + 0.25)^2$ or 0.35^2 seen A1 cao (no 'n' or 'n-1') divisors)M1 for $(0.05 + 0.05 + 0.25)^2$ or 0.35^2 seen A1 cao (no 'n' or 'n-1') divisors)5	(**)			
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$E(X^{2}) = 0 \times 0.55 + 1 \times 0.1 + 4 \times 0.05 + 9 \times 0.05 + 16 \times 0.25$ = 0 + 0.1 + 0.2 + 0.45 + 4 = (4.75) Var(X) = 'their' 4.75 - 1.35^{2} = 2.9275 awfw (2.9275 - 2.93) Var(X) = 'their' 4.75 - 1.35^{2} = 2.9275 awfw (2.9275 - 2.93) (iii) P(At least 2 both times) = (0.05+0.05+0.25)^{2} = 0.1225 o.e. (iii) P(At least 2 both times) = (0.05+0.05+0.25)^{2} = 0.1225 o.e. Al cao (no 'n' or 'n-1' divisors) 5			M1 for $\Sigma r^2 p$ (at least 3	
$\begin{array}{c c} = & 0 + 0.1 + 0.2 + 0.45 + 4 \\ = & (4.75) \\ Var(X) = & (their)^{2} 4.75 - 1.35^{2} = 2.9275 \text{ awfw} (2.9275 - 2.93) \\ Var(X) = & (their)^{2} 4.75 - 1.35^{2} = 2.9275 \text{ awfw} (2.9275 - 2.93) \\ A1 & (continuent on the content on the content of the content on the content of the cont$		$E(X^{2}) = 0 \times 0.55 + 1 \times 0.1 + 4 \times 0.05 + 9 \times 0.05 + 16 \times 0.25$	non zero terms correct)	
$ \begin{array}{c c} = (4.75) \\ Var(X) = `their` 4.75 - 1.35^2 = 2.9275 a wfw (2.9275 - 2.93) \\ \hline M1 dep for - their E(X)^2 \\ provided Var(X) > 0 \\ A1 cao (no `n' or `n-1' \\ divisors) \\ \hline 5 \end{array} $ $ \begin{array}{c c} \textbf{(iii)} \\ P(At least 2 both times) = (0.05 + 0.05 + 0.25)^2 = 0.1225 o.e. \\ M1 for (0.05 + 0.05 + 0.25)^2 = 0.1225 o.e. \\ A1 cao: a wfw (0.1225 - 0.123) or 49/400 \\ A1 cao: a wfw (0.1225 - 0.123) or 49/400 \\ \hline 2 \end{array} $		= 0 + 0.1 + 0.2 + 0.45 + 4		
Var(X) = 'their' $4.75 - 1.35^2 = 2.9275$ awfw $(2.9275 - 2.93)$ provided Var(X) > 0A1 cao (no 'n' or 'n-1' divisors)A1 cao (no 'n' or 'n-1' divisors)5(iii)P(At least 2 both times) = $(0.05+0.05+0.25)^2 = 0.1225$ o.e.M1 for $(0.05+0.05+0.25)^2$ or 0.35^2 seen A1 cao: awfw $(0.1225 - 0.123)$ or $49/400$ 2		= (4.75)	M1dep for – their $E(X)^2$	
Var(X) = 'their' $4.75 - 1.35^2 = 2.9275$ awtw $(2.9275 - 2.93)$ A1 cao (no 'n' or 'n-1' divisors) (iii) P(At least 2 both times) = $(0.05+0.05+0.25)^2 = 0.1225$ o.e. M1 for $(0.05+0.05+0.25)^2$ or 0.35^2 seen A1 cao: awfw $(0.1225 - 0.123)$ or $49/400$ 2			provided Var(X) > 0	
In case (nor nor nor nor nor nor nor nor nor nor		$\operatorname{var}(\mathbf{X}) = \operatorname{their} 4.75 - 1.35^2 = 2.9275 \text{ awfw} (2.9275 - 2.93)$	A1 cao (no 'n' or 'n-1'	
(iii) P(At least 2 both times) = $(0.05+0.05+0.25)^2 = 0.1225$ o.e. M1 for $(0.05+0.05+0.25)^2$ or 0.35^2 seen M1 for $(0.1225 - 0.1225)^2$ or $49/400$			divisors)	
(iii) P(At least 2 both times) = $(0.05+0.05+0.25)^2 = 0.1225$ o.e. M1 for $(0.05+0.05+0.25)^2$ or 0.35^2 seen A1cao: awfw $(0.1225 - 0.123)$ or $49/400$ 2			,	5
or 0.35^2 seen A1cao: awfw (0.1225 - 0.123) or $49/400$ 2	(iii)	P(At least 2 both times) = $(0.05+0.05+0.25)^2 = 0.1225$ o.e.	M1 for $(0.05+0.05+0.25)^2$	
$\begin{array}{c c} A1cao. awiw (0.1225 - \\ 0.123) \text{ or } 49/400 \end{array} = 2 \end{array}$			or 0.35° seen	
			0.123) or 49/400	2

TOTAL	8
IOTH	0

Q4	$X \sim B(50, 0.03)$		
(1)	(A) $P(X = 1) = {\binom{50}{1}} \times 0.03 \times 0.97^{49} = 0.3372$	M1 0.03×0.97^{49} or $0.0067(4)$	
		M1 $\binom{50}{1} \times pq^{49}$ (p+q	
		=1) A1 CAO (awfw 0. 337 to 0. 3372)	3
	(B) $P(X = 0) = 0.97^{50} = 0.2181$ P(X > 1) = 1 - 0.2181 - 0.3372 = 0.4447	or 0.34(2s.f.) or 0.34(2d.p.) but not just 0.34	
	$(X \times I) = 1 0.2101 0.3372 = 0.1117$	B1 for 0.97^{50} or 0.2181 (awfw 0.218 to 0.2181) M1 for 1 - ('their' p (X = 0) + 'their' p(X = 1))	3
		must have both probabilities A1 CAO (awfw 0.4447 to 0.445)	
(ii)	Expected number = $np = 240 \times 0.3372 = 80.88 - 80.93 = (81)$ Condone 240 × 0.34 = 81.6 = (82) but for M1 Alf.t.	M1 for 240×prob (A) A1FT	2
		TOTAL	8
Q5 (i)	P(R) × P(L) = $0.36 \times 0.25 = 0.09 \neq P(R \cap L)$ Not equal so not independent. (Allow $0.36 \times 0.25 \neq 0.2$ or 0.09 ≠ 0.2 or $\neq p(R \cap L)$ so not independent)	M1 for 0.36×0.25 or 0.09 seen A1 (numerical justification needed)	2
(ii)	R L	G1 for two overlapping circles labelled	
	.16 (0.2) 0.05	G1 for 0.2 and either 0.16 <i>or</i> 0.05 in the correct places	
	0.59	G1 for all 4 correct probs in the correct places (including the 0.59)	3
		The last two G marks are independent of the labels	
(111)	$P(L \mid R) = \frac{P(L \cap R)}{P(R)} = \frac{0.2}{0.36} = \frac{5}{9} = 0.556 \text{ (awrt 0.56)}$	M1 for 0.2/0.36 o.e. A1 cao	
	This is the probability that Anna is late given that it is raining. (must be in context) Condone 'if ' or 'when' or 'on a rainy day' for 'given that' but <u>not</u> the words 'and' or 'because' or 'due to'	E1 (indep of M1A1) Order/structure <u>must</u> be correct i.e. no reverse statement	3
		ΤΟΤΑΙ	0
		IOTAL	ð

Section B

Q6	Median = 4.06 – 4.075 (inclusive)	B1cao	
(1)	$Q_1 = 3.8$ $Q_3 = 4.3$	B1 for Q_1 (cao) B1 for Q_3 (cao)	
	Inter-quartile range = $4.3 - 3.8 = 0.5$	B1 ft for IQR must be using t-values not locations to earn this mark	4
(ii)	Lower limit ' their $3.8' - 1.5 \times$ 'their $0.5' = (3.05)$ Upper limit ' their $4.3' + 1.5 \times$ 'their $0.5' = (5.05)$ Very few if any temperatures <u>below 3.05 (but not zero)</u> None <u>above 5.05</u> 'So few, if any outliers' scores SC1	B1ft: must have -1.5 B1ft: must have +1.5 E1ft dep on -1.5 and Q_1 E1ft dep on+1.5 and Q_3 Again, must be using t- values NOT locations to	4
(iii)	Valid argument such as 'Probably not, because there is nothing to suggest that they are not genuine data items; (they do not appear to form a separate pool of data.') Accept: exclude outlier – 'measuring equipment was wrong' or 'there was a power cut' or ref to hot / cold day [Allow suitable valid alternative arguments]	E1	1
(iv)	Missing frequencies 25, 125, 50	B1, B1, B1 (all cao)	3
(v)	$Mean = (3.2 \times 25 + 3.6 \times 125 + 4.0 \times 243 + 4.4 \times 157 + 4.8 \times 50)/600$ $= 2432.8/600 = 4.05(47)$	M1 for at least 4 midpoints correct and being used in attempt to find $\sum ft$	2
		A1cao: awfw (4.05 – 4.055) ISW or rounding	
(vi)	New mean = $1.8 \times$ 'their $4.05(47)$ ' + $32 = 39.29(84)$ to 39.3 New s = 1.8×0.379 = 0.682	B1 FT M1 for 1.8 × 0.379 A1 CAO awfw (0.68 – 0.6822)	3
		TOTAL	17

Q7 (i)	$X \sim B(10, 0.8)$ (A) Either $P(X = 8) = {10 \choose 8} \times 0.8^8 \times 0.2^2 = 0.3020$ (awrt) or $P(X = 8) = P(X \le 8) - P(X \le 7)$ = 0.6242 - 0.3222 = 0.3020 (B) Either $P(X \ge 8) = 1 - P(X \le 7)$ = 1 - 0.3222 = 0.6778 or $P(X \ge 8) = P(X = 8) + P(X = 9) + P(X = 10)$ = 0.3020 + 0.2684 + 0.1074 = 0.6778	M1 $0.8^8 \times 0.2^2$ or 0.00671 M1 $\binom{10}{8} \times p^8 q^2$; (p+q =1) Or 45 × $p^8 q^2$; (p+q=1) A1 CAO (0.302) not 0.3 OR: M2 for 0.6242 – 0.3222 A1 CAO M1 for 1 – 0.3222 (s.o.i.) A1 CAO awfw 0.677 – 0.678 or M1 for sum of 'their' p(X=8) plus correct expressions for p(x=9) and p(X=10) A1 CAO awfw 0.677 – 0.678	3
(ii)	Let $X \sim B(18, p)$ Let p = probability of delivery (within 24 hours) (for population) H ₀ : $p = 0.8$ H ₁ : $p < 0.8$ P($X \le 12$) = 0.1329 > 5% ref: [pp =0.0816]	 B1 for definition of p B1 for H₀ B1 for H₁ M1 for probability 0.1329 M1dep strictly for comparison of 0.1329 with 5% (seen or clearly implied) 	
	So not enough evidence to reject H ₀ Conclude that there is not enough evidence to indicate that less than 80% of orders will be delivered within 24 hours Note: use of critical region method scores M1 for region {0,1,2,,9, 10} M1dep for 12 does not lie in critical region then A1dep E1dep as per scheme	Aldep on both M's Eldep on M1,M1,A1 for conclusion in context	7

(iii)	Let $X \sim B(18, 0.8)$		
(111)	$H_1: p \neq 0.8$	B1 for H_1	
	LOWER TAIL		
	$P(X \le 10) = 0.0163 < 2.5\%$	B1 for 0.0163 or 0.0513	
	$P(X \le 11) = 0.0513 > 2.5\%$	seen	
		M1dep for either correct comparison with 2.5% (not 5%) (seen or clearly implied)	
		A1dep for correct lower tail CR (must have zero)	
	UPPER TAIL $P(X \ge 17) = 1 - P(X \le 16) = 1 - 0.9009 = 0.0991 > 2.5\%$ $P(X > 18) = 1 - P(X \le 17) = 1 - 0.9820 = 0.0180 \le 2.5\%$	B1 for 0.0991 or 0.0180 seen	
		M1dep for either correct	
	So critical region is $\{\underline{0}, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 18\}$ o.e.	comparison with 2.5%	
	Condone $X \le 10$ and $X \ge 18$ or $X = 18$ but <u>not</u> $p(X \le 10)$ and	implied)	
	$p(X \ge 18)$	• ´	
	Correct CR without supportive working scores SC2 max after	A1dep for correct upper	
	the 1 st D1 (SC1 for each fully correct toil of CD)	tall CK	7
	the 1 B1 (SC1 for each fully correct tall of CK)		,
		TOTAL	19

4766 Statistics 1 (G241 Z1)

General Comments

The level of difficulty of the paper appeared to be entirely appropriate for the candidates with a good range of marks obtained. It was very pleasing to note the performance of the more able candidates who scored highly on all questions. The presentation of work was good in the majority of cases.

Most candidates supported their numerical answers with appropriate explanations and working although some rounding errors were noted. The possible exception was in question 7 where the procedure for distinguishing between hypotheses was not always clear and where the construction of the critical region was occasionally sketchy. There was not much evidence of the efficient use of statistical calculations on a calculator with most candidates (even the most able) preferring to commit all the stages of the calculation to paper.

Weaker candidates often scored a significant proportion of their marks from the calculation of E(X) and Var (X) in question 3 and from the use of the cumulative frequency curve in question 6. Particularly amongst lower scoring candidates there was evidence of the use of point probabilities in question 7, possibly more so than in very recent papers.

Comments on Individual Questions

Few candidates scored full marks on this question. Many found the mean as 0.75 but omitted the units. A small number of candidates divided by 1000 not 10000 whilst a few found $\sum fx$ as 5110 (the value of $\sum x$). Many struggled to find the standard deviation correctly with errors including the use of $\sum fx2$ as 70000, 25010000 or 29250000, or division by 10000 instead of 9999 although this error was less frequent than in the summer. There were a lot of answers around 50.2 from obviously incorrect working.

Fully correct answers to part (ii) were rare. There were many answers involving 50 - 50 - 20 = 20

 $\frac{50}{10000} \times \frac{50}{10000} + \frac{20}{10000} \times \frac{20}{10000}$ (with replacement) instead of the correct

 $\frac{50}{10000} \times \frac{49}{9999} + \frac{20}{10000} \times \frac{19}{9999}$ (without replacement) whilst others wrote down the correct probability terms for two £10 prizes and for two £100 prizes but then failed to

perform the necessary addition in order to gain the full marks. A small number attempted to use P (A or B) = P (A) + P (B) – P (A and B) or similar with a value for P(A and B).

Part (i) was often answered well although some candidates gave $(1/6)^6$ as the answer whilst others calculated 6! = 720 but failed to convert it into the correct probability. Part (ii) did not produce the same success with wrong answers including 6/20, 1/120, 20/120. Others found 6C_3 as 20 but then failed to use it correctly sometimes even using it as part of a binomial expression. Those using $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4}$ often forgot this could be arranged in 3! ways. 3 There were many excellent answers to parts (i) and (ii) even from the weaker candidates. The main error was to omit the subtraction of 1.352 in attempting to find the variance. Some of the weaker candidates squared the probabilities instead of r.

> Candidates found part (iii) much more taxing with a substantial number not obtaining 0.35; of those that did, few went on to reach 0.35². Some candidates made very heavy weather of this often failing to realise that they could just add the probabilities of 2, 3 and 4 to give the 0.35, for each occasion. Those who did often left it as this answer and failed to square it. Some calculated 1- 0.65^2 instead of $(1 - 0.65)^2$. Some considered the individual outcomes but apart from one or two they did not have all nine terms. Generally they had 0.052 +0.052+0.252. The other common wrong answer was $(0.6875)^2$. Some candidates multiplied by 2 instead of squaring 0.35.

> Some tried to tackle the problem by complements, believing that $p(X \ge 2 \text{ on both})$ occasions) = $1 - p \{0, 0 \text{ or } 1, 0 \text{ or } 1, 1\}$. Very few realised that if they went down this protracted route then what was required was 1 - p{ 0,0 or 0, 1 or 0,2 or 0,3 or 0,4 or 1,0 or 1,1 or 1,2 or 1,3 or 1,4 or 2,0 or 2,1or 3,0 or 3,1 or 4,0 or 4,1

- The stronger candidates regularly scored full marks on this question. Otherwise the 4 main errors in part (i) were the omission of ${}^{50}C_1$ or a miscalculation of a correct binomial expression. Attempts at part (ii) were less successful with a number of answers given as 1 - P(X = 0) or as 1 - P(X = 1) instead of $1 - P(X \le 1)$. Most candidates gave the expectation correctly as 240 \times P(X = 1) although some still insisted in rounding their answer to an integer. There was the very occasional use of 50 or 12000 instead of 240.
- 5 Although a number of candidates scored full marks, there were some very mixed responses to this question. In part (i) the stronger candidates gave clear and precise reasons as to why the events were not independent either from comparing $P(R \mid L)$ with P(R), or by comparing P(R and L) with P(R) \times P(L). Others did not make the comparison clear, or compared P(R|L) with P(L), or having found that P(R and L)was not equal to $P(R) \times P(L)$ said that the events were independent.

The Venn diagram in part (ii) was often poorly answered with probabilities of 0.36, 0.2. 0.25 and 0.19 for the four regions common instead of the correct 0.16, 0.2, 0.05 and 0.59. Another less common error was to replace the correct probability of 0.59 with 0.39 or even 0.41.

Part (iii) produced many correct answers alongside errors such as 0.2/0.25 and 0.25/0.36. Most candidates understood that the expression represented a conditional probability but some failed to give an explanation in context.

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6 There were many very good answers to this question with most candidates scoring a good proportion of the marks. It was decided that it would be fairer to candidates to award one extra mark in part (i) and one fewer in (iii). Virtually all used a correct method in part (i) to find the median. A common mistake was to write 4.7 for 4.07 and IQR with the occasional misread from the diagram. There was less success with part (ii) with answers often involving the median or a multiple of the IQR other than 1.5. Not all candidates appreciated the fact one of the boundaries for the outliers (3.05 and 5.05) lay within the data range and the other outside it.

In part (iii) only a few candidates stated that the outlier could be a valid data item but other sensible explanations were seen. The frequency table was often completed correctly and most candidates attempted to use the interval midpoints to estimate the mean with varying degrees of accuracy. The estimate of the mean in degrees Fahrenheit was well answered but the addition of 32 was a common error in attempts to find the standard deviation. Some started all over again causing them to waste time and effort by changing all the mid points to Fahrenheit. Invariably, errors occurred along the way.

There were some superb answers to this question with explanations showing a clear understanding of the methods involved. Many candidates, however, struggled with the hypothesis testing and critical region with some scoring marks (if any) only for the initial probabilities.

In part (i) (A) the probability that exactly 8 orders were delivered was usually tackled sensibly either by use of tables or from a binomial expression. The main errors were the use of 1 - 0.6242 or the omission of ${}^{10}C_8$. Answers to part (B) were less successful with the omission of P(X = 8) in summing probabilities, 1 - P(X = 8) or $1 - P(X \le 8)$ being common mistakes.

In part (ii) many candidates did not define p correctly or omitted it; there also remain errors in the notation used such as $H_0 = 0.8$ or H_0 : P(X) = 0.8. The use of point probabilities was the major error in the hypothesis test; other mistakes included the sole use of P (X \le 11) = 0.0513 in attempting to distinguish between the two hypotheses and the lack of a conclusion in context.

Attempts at finding the critical region in part (iii) were spoilt by a variety of errors. These included a frequent use of point probabilities, a comparison with 0.05 instead of 0.025, not stating any comparison, a lower critical region omitting 0 and an upper critical region including 17. Some candidates thought they were still testing 12 packets but using a two-tailed test.

Throughout parts (ii) and (iii) many candidates were not precise with their notation by not distinguishing clearly between <, \leq and =, for example it was fairly common to see P(X = 12) = 0.1329 instead of P (X \leq 12) = 0.1329 which was then clarified by a written explanation or a diagram. Candidates who tried to answer the hypothesis test using line diagrams or bar charts were often imprecise in their statistical arguments. It is important that they back up their diagrams with clear references to tail probabilities and make it 100% clear which values are in the critical region.

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