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1) $n = 100$, $\Sigma x = 24940$

$\Sigma x^2 = 6240780$

i) $\bar{x} = \frac{\Sigma x}{n} = \frac{24940}{100} = 249.4$

$sd_x = \sqrt{\frac{\Sigma x^2 - n\bar{x}^2}{n-1}}$

$sd_x = \sqrt{\frac{6240780 - 100 \times 249.4^2}{99}}$

$sd_x = 14.5$ to 3 s.f.

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ii) $y = 0.9x - 15$

$\bar{y} = 0.9\bar{x} - 15$

$\bar{y} = 0.9 \times 249.4 - 15$

$\bar{y} = 209.46 = 209$ to 3 s.f.

$sd_y = 0.9 \times sd_x$

$= 0.9 \times 14.5$

$= 13.05$

$= 13.1$ to 3 s.f.

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2) 5 men, 5 women, 3 prizes

i) Number of ways of choosing 2 women and 1 man

$= {}^5C_2 \times {}^5C_1$

$= 10 \times 5 = 50$

Number of ways of choosing 3 people

$= {}^{10}C_3 = 120$

$Prob(2W, 1M) = \frac{50}{120} = \frac{5}{12}$

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ii) $P(\text{At least 3 (2W, 1M)})$

$= P(\text{Exactly 3}) + P(\text{Exactly 4})$

$= 4 \times \left(\frac{5}{12}\right)^3 \times \frac{7}{12} + \left(\frac{5}{12}\right)^4$

$= 0.1989$

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3) $X \sim B(50, 0.1)$

i) $P(X = 5) = {}^{50}C_5 \times 0.1^5 \times 0.9^{45}$

$= 0.1849$

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ii) $X \sim B(20, 0.1)$

$P(X \geq 1) = 1 - P(X = 0)$

$= 1 - 0.9^{20}$

$= 0.8784$

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iii) 48×0.8784

$= 42.16$

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4) i) For $X = 15$ the three containing £5 must be selected

$$P(X=15) = \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4}$$

$$= \frac{1}{20} = 0.05$$

ii)

r^2	225	1020100	4020025	9000000
r	15	1010	2005	3000
$P(x=r)$	0.05	0.45	0.45	0.05

$$E(x) = 0.05 \times 15 + 0.45 \times 1010 + 0.45 \times 2005 + 0.05 \times 3000$$

$$E(x) = £1507.50$$

$$E(x^2) = 0.05 \times 225 + 0.45 \times 1020100 + 0.45 \times 4020025 + 0.05 \times 9000000$$

$$E(x^2) = 2,718,067.5$$

$$Var(x) = E(x^2) - (E(X))^2$$

$$= 2,718,067.5 - 1507.5^2$$

$$Var(x) = 445,511.25$$

5) i) $p = 0.5$ because if a randomly chosen person simply guessed they would have a 50% chance of success.

ii) $p > 0.5$ because there is no reason a randomly chosen person should do worse than by guessing, so the alternative is to do better.

iii) $X \sim B(20, 0.5)$

$$P(X \geq 13) = 1 - P(X \leq 12)$$

$$= 1 - 0.8684$$

$$= 0.1316 > 5\%$$

Therefore accept $H_0: p = 0.5$

There is not sufficient evidence to support the view that people do any better than they would by guessing.

6) Median is item $\frac{25+1}{2} = 13^{th}$

i) Median = 3.32 kg

LQ identify median of lower 12 halfway between 6th and 7th

LQ = 2.83

UQ halfway between 19th and 20th

UQ = 3.71

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6i cont) $IQR = UQ - LQ$

$= 3.71 - 2.83$

$= 0.88$ kg

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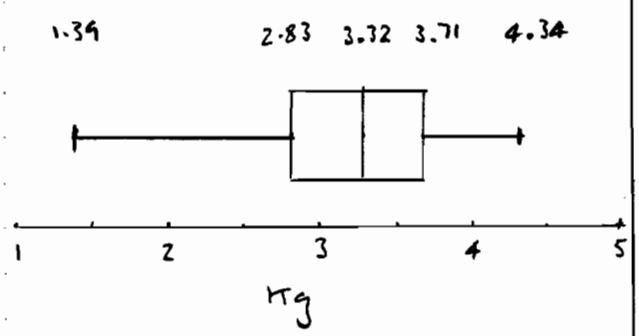
ii)



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iii) Outliers above

$UQ + 1.5 \times IQR$

$= 3.71 + 1.5 \times 0.88 = 5.03$

so no outliers at top end

Outliers below

$LQ - 1.5 \times IQR$

$= 2.83 - 1.5 \times 0.88 = 1.51$

1.39 is the only data item below 1.51 so there is exactly one outlier

Premature babies are sometimes born at less than 1.39 kg so it is reasonable to keep data item

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iv) Median = 3.5 kg

$IQR = UQ - LQ = 3.84 - 3.12$

$IQR = 0.72$ kg

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v) Males have a greater median birthweight than females

Male birthweights are slightly more consistent than female birthweights as they have a smaller IQR.

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vi) Males with birthweight > 4.34 kg appear to be 8.

so $P(\text{Both} > 4.34) = \frac{8}{200} \times \frac{7}{199}$

≈ 0.0014

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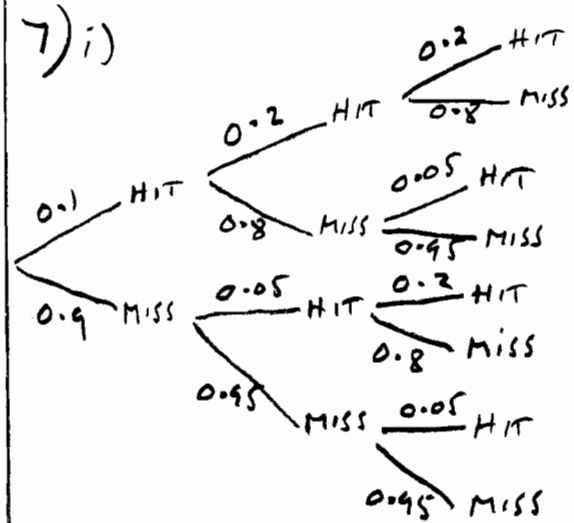
7) i)



ii) A) $P(\text{At least 1 HIT}) = 1 - P(3 \text{ MISSES})$

$= 1 - 0.9 \times 0.95 \times 0.95$

$= 0.18775$

$= 0.188$ to 3 s.f.

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7 ii B) P(exactly 1 hit)

$= P(HMM) + P(MHM) + P(MMH)$

$= 0.1 \times 0.8 \times 0.95$

$+ 0.9 \times 0.05 \times 0.8$

$+ 0.9 \times 0.95 \times 0.05$

$= 0.15475$

$= 0.155$ to 3 s.f.

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iii) P(exactly 1 hit / at least 1 hit)

$= \dfrac{P(\text{exactly 1 hit} \cap \text{at least 1 hit})}{P(\text{at least 1 hit})}$

$= \dfrac{0.15475}{0.18775}$

$= 0.8242$

$= 0.824$ to 3 s.f.

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iv) Prob(3 Hits in total)

$= P(HHH) + P(MMMHHH)$

$= 0.1 \times 0.2 \times 0.2$

$+ 0.9 \times 0.95 \times 0.95$
$\times 0.05 \times 0.2 \times 0.2$

$= 0.0056245$

$= 0.0056$ to 2 s.f.
