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1 i) $\frac{30}{50} \times \frac{29}{49} \times \frac{28}{48} = 0.2071$

ii) P(At least 1 of each)

$= 1 - P(\text{All blue}) - P(\text{All red})$

$P(\text{All red}) = \frac{20}{50} \times \frac{19}{49} \times \frac{18}{48}$

$= 0.0582$

$1 - 0.2071 - 0.0582$

$= 0.7347$

2 i) $5C_3 \times 9C_3 = 10 \times 84 = 840$

ii) No of ways of selecting 6 from $9+5=14$

$14C_6 = 3003$

P(Chooses 3 from each section)

$= \frac{840}{3003} = 0.2797$

3 i) $X \sim B(30, 0.85)$

$P(X=29) = {}^{30}C_{29} \times 0.85^{29} \times 0.15$

$= 0.0404$

ii) $P(\text{At least } 29) = P(29) + P(30)$

$P(X=30) = 0.85^{30} = 0.0076$

$P(X \geq 29) = 0.0404 + 0.0076$

$= 0.0480$

iii) $10 \times 0.048 = 0.48$

Expected number of samples with at least 29 on hold $= 0.48$

4 i A) $0.92 \times 0.92 \times 0.08$

$= 0.0677$

B) $P(\text{second}) = 0.92 \times 0.08$

$= 0.0736$

$P(\text{second or third}) = 0.0736 + 0.0677$

$= 0.1413$

4 ii) $P(\text{At least } 1) = 1 - P(\text{None})$

$= 1 - 0.92^{20}$

$= 0.8113$

5. $X \sim B(18, 0.05)$

$H_0: p = 0.05$, $H_1: p > 0.05$

p is prob a frame is faulty

$E(X) = 0.9$ $P(X \geq 4) = 1 - P(X \leq 3)$

$= 1 - 0.9891$

$= 0.0109 < 5\%$

There is sufficient evidence to reject H_0 and accept H_1

Conclude that proportion of faulty frames has increased above 5%

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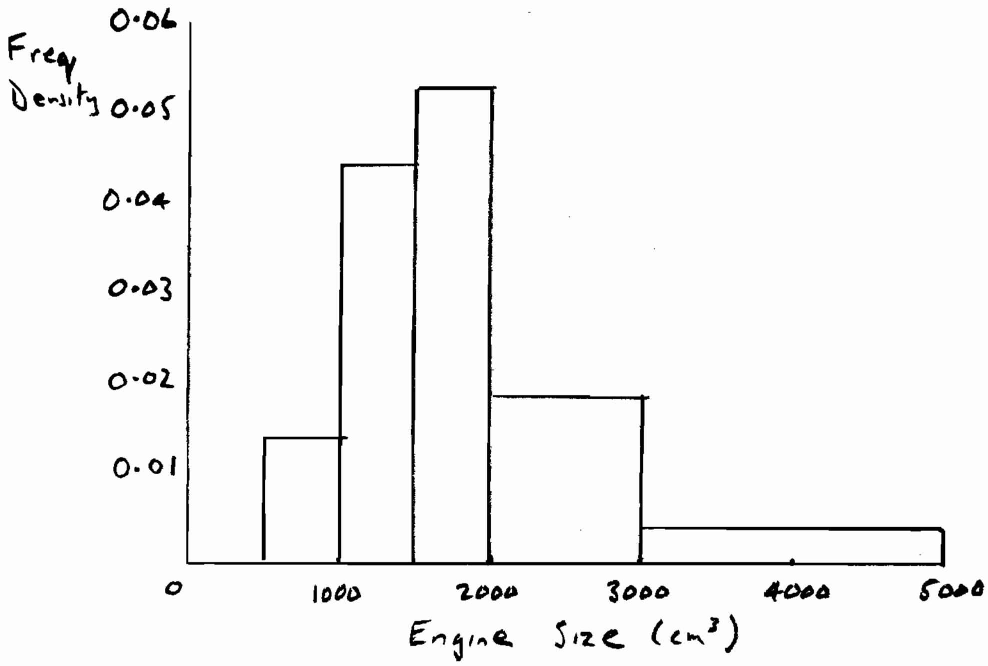
6.

i)

Engine Size	$500 \le x \le 1000$	$1000 < x \le 1500$	$1500 < x \le 2000$	$2000 < x \le 3000$	$3000 < x \le 5000$
Frequency	7	22	26	18	7
Freq. Density	0·014	0·044	0.052	0.018	0.0035



ii) Midrange = 2750 depends on lowest and data items adding to 5500. Possible but unlikely

iii) $\bar{x}$ estimate

freq	midpt	f × md
7	750	5250
22	1250	27500
26	1750	45500
18	2500	45000
7	4000	28000
80		151250

$\bar{x}$ estimate $= \dfrac{151250}{80} = 1891$ cc

$$s.d = \sqrt{\frac{\Sigma f x^2 - n\bar{x}^2}{n-1}}$$

estimate s.d.

$\Sigma f x^2 = 7 \times 750^2 + 22 \times 1250^2 + 26 \times 1750^2 + 18 \times 2500^2 + 7 \times 4000^2$

$= 342,437,500$

sd estimate $= \sqrt{\dfrac{342,437,500 - 80 \times 1891^2}{79}}$

$= 845$ cc

$\bar{x}$ and s.d are estimates as data is grouped with individual data items unknown

iv) Outliers $\bar{x} \pm 2 \times$ s.d.

$1891 + 2 \times 845 = 3581$

$1891 - 2 \times 845 = 201$

It is likely that their are outliers at the top end in the group $3000 < x \le 5000$ cc

There are no outliers at the bottom.

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6 v) Proportion of cars over 2000cc

$$= \frac{18+7}{80} = \frac{25}{80}$$

Money raised $= 2.5 \times 10^6 \times \frac{25}{80} \times 1000$

$= £781,250,000$

6vi) More people would buy smaller engined cars to avoid this duty, thereby reducing the amount of large engined cars sold

7 i) $P(X=0) = 0.4 \times 0.5^4$
$= 0.025$

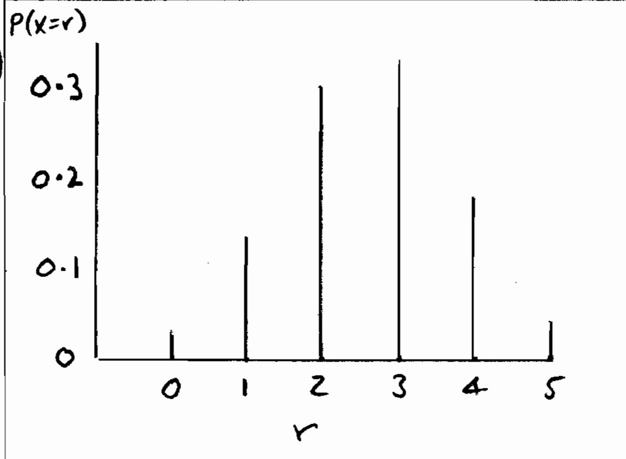
ii) $P(X=1)$
$= P(\text{Biased Head and others tails})$
$+ P(\text{Biased Tail and one head in others})$

$= 0.6 \times 0.5^4 + 0.4 \times {}^4C_1 \times 0.5 \times 0.5^3$

$= 0.6 \times 0.5^4 + 0.4 \times 4 \times 0.5^4$

$= 0.1375$

iii)



7 iv) Very slight negative skew but almost symmetrical.

7 v)

$E(X) = 0.025 \times 0 + 0.1375 \times 1$
$+ 0.3 \times 2 + 0.325 \times 3$
$+ 0.175 \times 4 + 0.0375 \times 5$

$\underline{E(X) = 2.6}$

$E(X^2) = 0.025 \times 0^2 + 0.1375 \times 1^2$
$+ 0.3 \times 2^2 + 0.325 \times 3^2$
$+ 0.175 \times 4^2 + 0.0375 \times 5^2$

$E(X^2) = 8$

$Var(X) = E(X^2) - (E(X))^2$

$= 8 - 2.6^2 = 1.24$

7 vi)

Could get	0, 0, 3	× 3
	0, 1, 2	× 6
	1, 1, 1	× 1

These multiples refer to how many ways the outcomes can be arranged eg |3,0,0 | 0,3,0 | 0,0,3|

Prob (Total heads is 3)

$= 0.025^2 \times 0.325 \times 3$

$+ 0.025 \times 0.1375 \times 0.3 \times 6$

$+ 0.1375^3 \times 1$

$= 0.0094$

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