

MEI STATISTICS 2 JANUARY 2009 Q2 POISSON

2 i) $X \sim B(5000, \frac{1}{10000})$

ii) n is large, p is small

$$\text{Mean } np = 0.5 = \lambda$$

iii) Approximating with $X \sim \text{Poisson}(0.5)$

$$\begin{aligned} P(X \geq 1) &= 1 - P(X=0) \\ &= 1 - e^{-0.5} \\ &= 0.3935 \end{aligned}$$

iv) $X \sim B(20, 0.3935)$

$$\begin{aligned} P(X=9) &= {}^{20}C_9 \times 0.3935^9 \times 0.6065^{11} \\ &= 0.1552 \end{aligned}$$

v) $E(x) = np = 20 \times 0.3935 = 7.87$

vi) Mean $\bar{x} = \frac{0 \times 11 + 1 \times 7 + 2 \times 2}{20} = \frac{11}{20} = 0.55$

$$\begin{aligned} \text{Variance } s^2 &= \frac{\sum f_x^2 - n\bar{x}^2}{n-1} = \frac{7 \times 1^2 + 2 \times 2^2 - 20 \times 0.55^2}{19} \\ s^2 &= 0.471 \end{aligned}$$

vii) Yes since mean is close to variance $0.55 \approx 0.471$

Also expected frequency for 'at least one' is close to observed frequency 7.87 close to $7+2=9$