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1 i) Uniform average rate of occurence E1

Successive arrivals are independent E1

Suitable arguments for/against each assumption

Eg Rate of occurrence could vary depending on the weather (any reasonable assumption) E1 E1

must be in context

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1 ii)

x	0	1	2	3	4	5	>5
f	18	39	20	12	8	3	0

Sample mean $\bar{x} = \dfrac{18\times0 + 39\times1 + 20\times2 + 12\times3 + 8\times4 + 3\times5}{18+39+20+12+8+3}$

$$\bar{x} = \frac{162}{100} = 1.62$$

sample variance

$$s^2 = \frac{\sum fx^2 - n\bar{x}^2}{n-1} = \frac{430 - 100\times1.62^2}{99}$$

$$s^2 = 1.69$$

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1 iii) In a Poisson distribution the mean and variance are equal. Since $1.69 \approx 1.62$ the calculations in part (ii) support the use of the Poisson distribution to model this situation.

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1 iv) $X \sim \text{Poisson}(1.62)$

$$P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

$$P(X=2) = \frac{e^{-1.62} \times 1.62^2}{2!} = 0.2597$$

This probability is reasonably close to the observed relative frequency of 0.2

1 v) For 50 second periods

$$X \sim \text{Poisson}(1.62 \times 5)$$

$$X \sim \text{Poisson}(8.1)$$

$$\begin{aligned} P(X \geq 10) &= 1 - P(X \leq 9) \\ &= 1 - 0.7041 \\ &= 0.2959 \end{aligned}$$

1 vi) For time period of 1 hour

$$X \sim \text{Poisson}(1.62 \times 6 \times 60)$$

$$X \sim \text{Poisson}(583.2)$$

Approximate with

$$X \sim N(\underset{\mu}{583.2}, \underset{\sigma^2}{\sqrt{583.2}^2})$$

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1vi cont) Find $P(X \le 550.5)$

$$z = \frac{x-\mu}{\sigma}$$

$$z = \frac{550.5 - 583.2}{\sqrt{583.2}}$$

$$z = -1.354$$

$P(Z < -1.354) = 1 - P(Z < 1.354)$

$= 1 - 0.9121$

$= 0.0879$