MEI STATISTICS 2 JANUARY 2008 Q3 NORMAL

3 i) $X \sim N(56, 6.5^2)$

X 52.5 56 57.5

Z −0.539 0.231

$$z = \frac{x - \mu}{\sigma}$$

when $x = 57.5$ $\quad z = \dfrac{57.5 - 56}{6.5}$

$z = 0.2308$

when $x = 52.5$ $\quad z = \dfrac{52.5 - 56}{6.5}$

$z = -0.5385$

$P(52.5 < X < 57.5)$

$= P(-0.539 < Z < 0.231)$

$= \Phi(0.231) - (1 - \Phi(0.539))$

$= \Phi(0.231) + \Phi(0.539) - 1$

$= 0.5914 + 0.7050 - 1$

$= 0.2964$

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3 ii) Child $X \sim N(5.6, 6.5^2)$ $\qquad$ Adult $X \sim N(68, 10^2)$

$$z = \frac{x - \mu}{\sigma}$$

$z = \dfrac{62 - 56}{6.5} = 0.923$

$P(X < 62) = P(Z < 0.923)$

$= 0.8220$

$z = \dfrac{62 - 68}{10} = -0.6$

$P(X < 62) = P(Z < -0.6)$

$= 1 - P(Z < 0.6)$

$= 1 - 0.7257$

$= 0.2743$

3 ii cont)

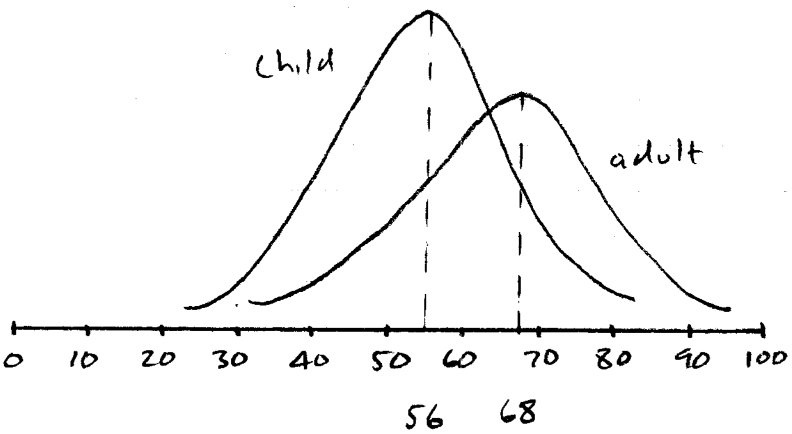
Prob (1 over 62 and 1 under 62)

$= P(\text{Adult over } 62 \cap \text{Child under } 62) + P(\text{Adult under } 62 \cap \text{Child over } 62)$

$= 0.7257 \times 0.8220 + 0.2743 \times 0.1780$

$= 0.6454$

3 iii)

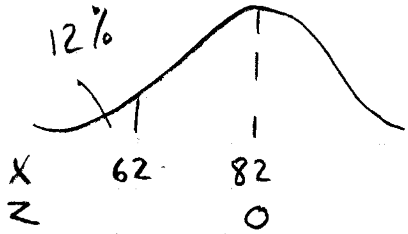


Note:

The area under a complete normal curve must equal 1

Therefore the one with the bigger standard deviation (more spread out) is not as tall as the other one

3 iv) $X \sim N(82, \sigma^2)$



At $x = 62$ with 12% below

$$z = -\Phi^{-1}(0.88) = -1.175$$

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3 iv) cont

$$z = \frac{x-\mu}{\sigma}$$

$$\Rightarrow \sigma z = x - \mu$$

$$\sigma = \frac{x-\mu}{z}$$

$$\sigma = \frac{62-82}{-1.175}$$

$$\sigma = 17.02$$