MEI STATISTICS 2 JANUARY 2009 Q3 NORMAL

3i) A) $X \sim N(115.3, 21.9^2)$

$P(X < 120)$

$= P(z < 0.215)$

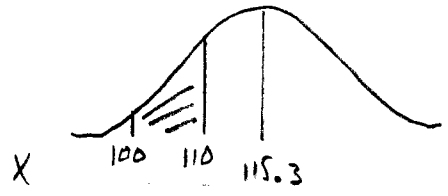
$= 0.5851$

$z = \frac{x - \mu}{\sigma}$

$z = \frac{120 - 115.3}{21.9}$

$z = 0.215$

B)



$z = \frac{100 - 115.3}{21.9} = -0.699$

$z = \frac{110 - 115.3}{21.9} = -0.242$

$P(100 < X < 110) = P(-0.699 < z < -0.242)$

$= P(0.242 < z < 0.699)$

$= \Phi(0.699) - \Phi(0.242)$

$= 0.7578 - 0.5956$

$= 0.1622$

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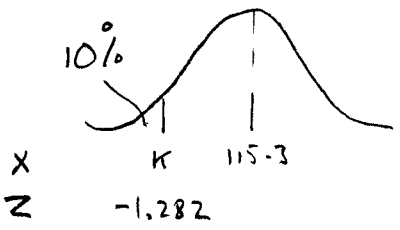
c)



At K $z = -\Phi^{-1}(0.9)$

$z = -1.282$

$z = \frac{K - \mu}{\sigma}$

$\sigma z = K - \mu$

$\sigma z + \mu = K$

$K = 21.9 \times (-1.282) + 115.3 = 87.22$

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3ii) $Y \sim N(\mu, \sigma^2)$

$P(Y < 180) = 0.7$, $P(Y < 140) = 0.15$

$y = 180$ $y = 140$

$z = \Phi^{-1}(0.7) = 0.5244$ $z = -\Phi^{-1}(0.85) = -1.036$

$$z = \frac{y - \mu}{\sigma} \Rightarrow \sigma z = y - \mu$$

Substituting known values

$$0.5244\sigma = 180 - \mu \quad (1)$$

$$-1.036\sigma = 140 - \mu \quad (2)$$

(1) - (2) $1.5604\sigma = 40$

$$\sigma = \frac{40}{1.5604} = 25.63$$

Substitute for σ in (1)

$0.5244 \times 25.63 = 180 - \mu$

$\mu = 180 - 0.5244 \times 25.63 = 166.56$

$\mu = 166.56$, $\sigma = 25.63$

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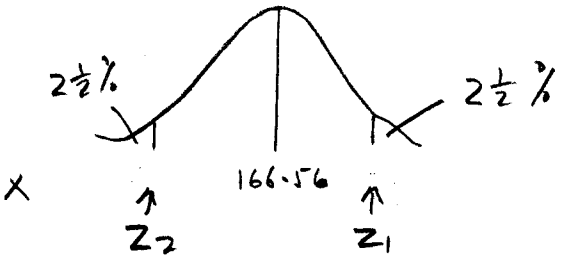
3iii) $Y \sim N(166.56, 25.63^2)$



$z_1 = \Phi^{-1}(0.975)$

$z_2 = -\Phi^{-1}(0.975$

$z_1 = 1.960$

$z_2 = -1.960$

$z = \frac{y-\mu}{\sigma} \Rightarrow \sigma z = y - \mu$

$\Rightarrow \sigma z + \mu = y$

when $z = 1.960$

$y = 1.960 \times 25.63 + 166.56 = 216.8$

when $z = -1.960$

$y = -1.960 \times 25.63 + 166.56 = 116.3$

$\Rightarrow P(116.3 < Y < 216.8) = 0.95$