

3 i)  $X \sim N(45.3, 11.5^2)$



$$Z = \frac{x - \mu}{\sigma}$$

$$Z = \frac{50 - 45.3}{11.5} = 0.409$$

$$P(X < 50) = P(Z < 0.409)$$

$$= 0.6586$$


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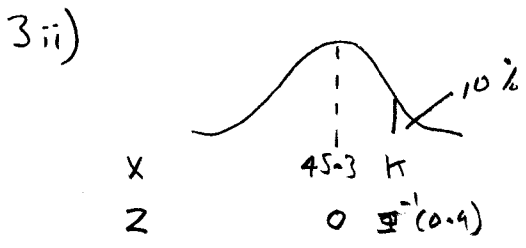
$$P(45.3 < X < 50)$$

$$= P(0 < Z < 0.409)$$

$$= 0.6586 - 0.5$$

$$= 0.1586$$


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At k where 10% score more

$$z = \Phi^{-1}(0.9) = 1.282$$

$$z = \frac{k - \mu}{\sigma}$$

$$\sigma z + \mu = k$$

$$k = 11.5 \times 1.282 + 45.3$$

$$k = 60.04$$



3iii)  $X \sim N(100, 15^2)$

Find  $P(110.5 < X < 111.5)$

$$Z = \frac{x - \mu}{\sigma}$$

$$= P(0.7 < Z < 0.767)$$

$$x = 111.5, \quad Z = \frac{111.5 - 100}{15}$$

$$Z = 0.767$$

$$= \Phi(0.767) - \Phi(0.7)$$

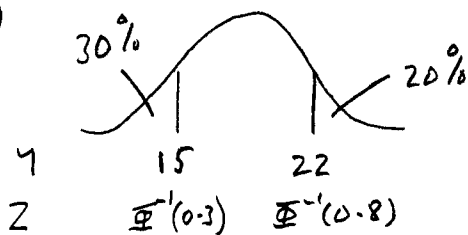
$$x = 110.5 \quad Z = \frac{110.5 - 100}{15}$$

$$= 0.7785 - 0.7580$$

$$Z = 0.7$$

$$= 0.0205$$

3iv)



when  $y = 15$ ,  $Z = -0.5244$

when  $y = 22$ ,  $Z = 0.8416$

$$\Phi^{-1}(0.3) = -\Phi^{-1}(0.7) = -0.5244$$

$$\Phi^{-1}(0.8) = 0.8416$$

$$Z = \frac{y - \mu}{\sigma}$$

$$\sigma Z = y - \mu$$

Substituting

$$\sigma \times (-0.5244) = 15 - \mu \quad (1)$$

$$\sigma \times 0.8416 = 22 - \mu \quad (2)$$

$$(2) - (1) \quad 1.366\sigma = 7 \quad \Rightarrow \quad \sigma = \frac{7}{1.366} = 5.124$$

Subst for  $\sigma$  in (2)  $5.124 \times 0.8416 = 22 - \mu$

$$\mu = 22 - 5.124 \times 0.8416 = 17.69$$

$$\mu = 17.69, \quad \sigma = 5.124$$

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