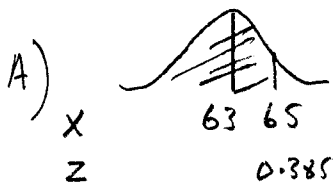3 i) $X \sim N(63, 5.2^2)$
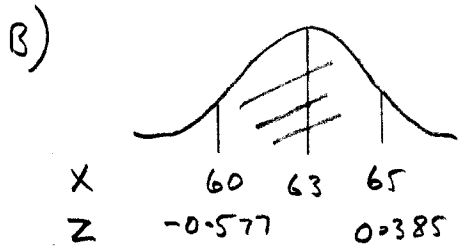
A)

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{65 - 63}{5.2} = 0.385$$

X   63 65
Z   0.385

$$P(X < 65) = P(z < 0.385)$$

$$= 0.6499$$

B)

$$z = \frac{60 - 63}{5.2} = -0.577$$

X   60 63 65
Z   -0.577   0.385

$$P(60 < X < 65) = P(-0.577 < z < 0.385)$$
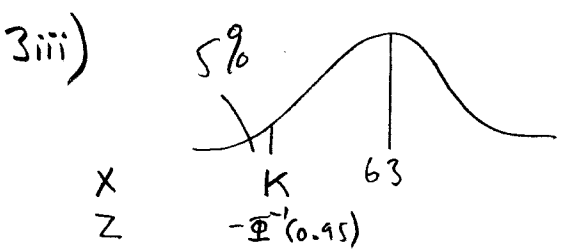
$$= P(z < 0.385) - P(z < -0.577)$$

$$= P(z < 0.385) - (1 - P(z < 0.577))$$

$$= \Phi(0.385) + \Phi(0.577) - 1$$

$$= 0.6499 + 0.7181 - 1$$

$$= 0.368$$

3 ii) $(0.368)^5 = 0.00675$

3 iii)

5%

X   K   63
Z $-\Phi^{-1}(0.95)$

At $x = k$

$$z = -\Phi^{-1}(0.95) = -1.645$$

$$z = \frac{x - \mu}{\sigma} \Rightarrow x = \sigma z + \mu$$

$$k = -1.645 \times 5.2 + 63 = 54.45 \text{ seconds}$$
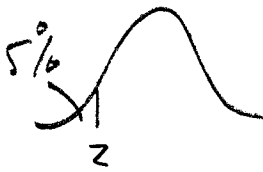
3iv) $H_0 : \mu = 63$ where μ is the mean time
taken on new course

$H_1 : \mu < 63$

$X \sim N(63, 5.2^2)$

For sample of 15, $X \sim N\left(63\left(\dfrac{5.2}{\sqrt{15}}\right)^2\right)$

5%

z

For 5% significance level test
at lower end

critical $z = -\Phi^{-1}(0.95) = -1.645$

$z = \dfrac{61.7 - 63}{\dfrac{5.2}{\sqrt{15}}} = -0.968$

Since $-1.645 < -0.968$ Accept H_0

There is insufficient evidence to suggest that the
mean time trial time is reduced on the new course

Conclude that mean remains at 63 minutes