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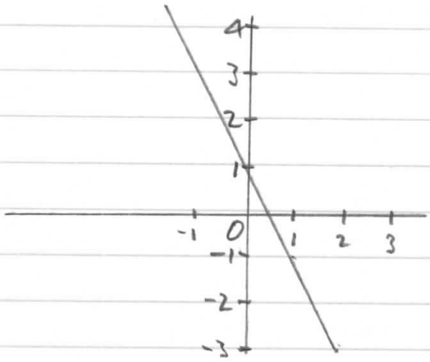
1)



2) $\left(1\frac{7}{9}\right)^{-\frac{1}{2}} = \left(\frac{16}{9}\right)^{-\frac{1}{2}}$

i)

$= \left(\frac{9}{16}\right)^{\frac{1}{2}} = \frac{3}{4}$

ii) $\frac{(6x^5y^2)^3}{18y^{10}}$

$= \frac{216x^{15}y^6}{18y^{10}}$

$= \frac{12x^{15}}{y^4}$ or $12x^{15}y^{-4}$

3) $6 - x > 5(x-3)$

$6 - x > 5x - 15$

$6 + 15 > 5x + x$

$21 > 6x$

$\frac{21}{6} > x$

$x < \frac{7}{2}$

4) $2x + 5y = 5$ ①

$x - 2y = 4$ ②

from ② $x = 4 + 2y$

sub for x in ①

$2(4+2y) + 5y = 5$

$8 + 4y + 5y = 5$

$9y = 5 - 8$

$9y = -3$

$y = -\frac{3}{9}$

$y = -\frac{1}{3}$

Now $x = 4 + 2y$

$x = 4 - \frac{2}{3}$

$x = 3\frac{1}{3}$ or $\frac{10}{3}$

Solution $x = \frac{10}{3}$, $y = -\frac{1}{3}$

5) $(x+2)^2 + (y-3)^2 = 5$

i) radius $= \sqrt{5}$

centre $(-2, 3)$

ii) // Line of form

$5x + y = c$

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5ii) cont)

$$5x + y = c$$

Sub $(-2, 3)$

$$5(-2) + 3 = c$$

$$-10 + 3 = c$$

$$-7 = c$$

Line is $5x + y = -7$

or $y = -5x - 7$

6)

$$r = \sqrt{\frac{V}{a+b}}$$

$$r^2 = \frac{V}{a+b}$$

$$r^2(a+b) = V$$

$$r^2 a + r^2 b = V$$

$$r^2 b = V - r^2 a$$

$$b = \frac{V - r^2 a}{r^2}$$

7) i)

$$\frac{5 - 2\sqrt{7}}{3 + \sqrt{7}} \times \frac{3 - \sqrt{7}}{3 - \sqrt{7}}$$

$$= \frac{15 - 6\sqrt{7} - 5\sqrt{7} + 14}{3^2 - \sqrt{7}^2}$$

$$= \frac{29 - 11\sqrt{7}}{9 - 7}$$

$$= \frac{29 - 11\sqrt{7}}{2}$$

8) $(a + bx)^5$ constant 32

coeff of $x^3 = -1080$

$a^5 = 32 \Rightarrow a = \sqrt[5]{32}$

$a = 2$

Coeff of $x^3 =$

$${}^5C_3\, a^2 b^3 = -1080$$

$$10 \times 4 \times b^3 = -1080$$

$$b^3 = \frac{-108\cancel{0}}{4\cancel{0}}$$

$$b^3 = -27$$

$$b = \sqrt[3]{-27}$$

$$b = -3$$

$a = 2, \quad b = -3$

9) $n, \; n+1, \; n+2$

$$(n+2)^2 - n^2$$

$$= n^2 + 4n + 4 - n^2$$

$$= 4n + 4$$ difference

$$4n + 4 = 4(n+1)$$

which is four times middle one

7ii) at end of paper accidentally omitted

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Section B

10)

$A(3,3)$

$B(-2,-2)$

i) $C(5,-1)$

$|AB| = \sqrt{(3--2)^2+(3--2)^2}$

$= \sqrt{5^2+5^2} = \sqrt{50}$

$|BC| = \sqrt{(-2-5)^2+(-2--1)^2}$

$= \sqrt{(-7)^2+(-1)^2}$

$= \sqrt{49+1} = \sqrt{50}$

$\therefore AB = BC$

ii) Gradient of $AC = \frac{3--1}{3-5} = \frac{4}{-2}$

$= -2$

$\Rightarrow \perp$ line has gradient $+\frac{1}{2}$

Through $B(-2,-2)$

$y - y_1 = m(x - x_1)$

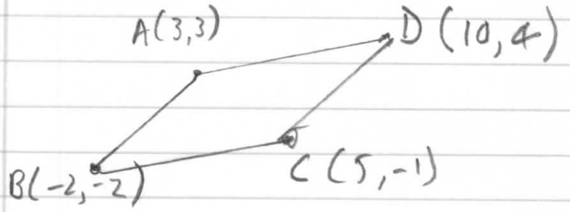
$y - -2 = \frac{1}{2}(x - -2)$

$y+2 = \frac{1}{2}(x+2)$

$y+2 = \frac{1}{2}x + 1$

$y = \frac{1}{2}x - 1$

iii)



Using congruent Δs

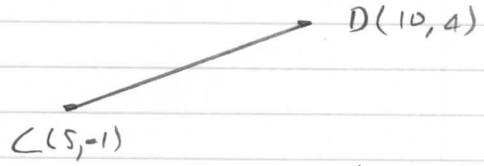
$D = (5+5, -1+5)$

$D(10,4)$

(Same changes as from B to A)

iv) $E(8, 3.8)$

Consider position in relation to CD

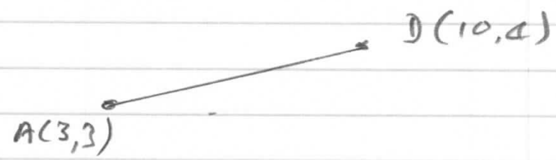


when $x = 8$, point on CD

is $\left(8, -1 + \frac{3}{5}(5)\right) = (8,2)$

so E is above line CD

Consider position in relation to AD



when $x = 8$, point on CD

is $\left(8, 3 + \frac{5}{7}(1)\right)$

$= \left(8, 3\frac{5}{7}\right)$ $\quad 7\overline{)5.000} = 0.714$

$\approx (8, 3.74)$

so $(8, 3.8)$ is above AD

and therefore outside

the rhombus

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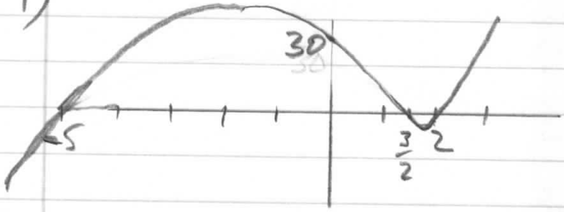
11) i) $f(x) = (x-2)(2x-3)(x+5)$



ii) $g(x) = (x+1)(2x+3)(x+8)$

(Since $g(x) = f(x+3)$)

$g(x) = (2x^2+5x+3)(x+8)$

$= 2x^3+5x^2+3x$
$+16x^2+40x+24$

$= 2x^3+21x^2+43x+24$

iii) $g(-2) = 2(-2)^3+21(-2)^2$
$+43(-2)+24$

$= -16+84-86+24$

$= 6$

so $x=-2$ is root of $g(x)=6$

$2x^3+21x^2+43x+24=6$

$2x^3+21x^2+43x+18=0$

$$\begin{array}{r|l} & 2x^2+17x+9 \\ x+2 & 2x^3+21x^2+43x+18 \\ & 2x^3+4x^2 \\ & \quad 17x^2+43x \\ & \quad 17x^2+34x \\ & \qquad 9x+18 \\ & \qquad 9x+18 \end{array}$$

$(x+2)(2x^2+17x+9)=0$

$x = \dfrac{-17 \pm \sqrt{17^2-4\times2\times9}}{4}$

$x = \dfrac{-17 \pm \sqrt{289-72}}{4}$

$x = \dfrac{-17 \pm \sqrt{217}}{4}$

12) i) $y = x^2+x+3$

$y = (x+\frac{1}{2})^2+3-\frac{1}{4}$

$y = (x+\frac{1}{2})^2+\frac{11}{4}$

Min value of $y = \frac{11}{4}$ so does not cross x-axis

ii) $y = x^2+x+3$ (1)

$y = 2x^2-3x-9$ (2)

Sub for y in (1)

$2x^2-3x-9 = x^2+x+3$

$2x^2-3x-9-x^2-x-3=0$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$\Rightarrow x=6$ or $x=-2$

$y = 6^2+6+3$
$= 45$

$y = (-2)^2+(-2)+3$
$= 4-2+3$
$= 5$

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12ii cont) Points of intersection

$(6, 45)$ and $(-2, 5)$

iii) $2x^3 - 3x - 9 - x^2 - x - k = 0$

$x^2 - 4x + (-9-k) = 0$

for no intersection $b^2 < 4ac$

$16 < 4 \times 1 \times (-9-k)$

$16 < -36 - 4k$

$4k < -36 - 16$

$4k < -52$

$k < \frac{-52}{4}$

$k < -13$

7ii) $\frac{12}{\sqrt{2}} + \sqrt{48}$

$= \frac{12}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + \sqrt{49 \times 2}$

$= \frac{12\sqrt{2}}{2} + 7\sqrt{2}$

$= 6\sqrt{2} + 7\sqrt{2}$

$= 13\sqrt{2}$

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