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1) $c = \sqrt{\frac{a+b}{2}}$

$\Rightarrow c^2 = \frac{a+b}{2}$

$\Rightarrow 2c^2 = a+b$

$\Rightarrow a = 2c^2 - b$

2) $\frac{5x-3}{2} < x+5$

$5x - 3 < 2x + 10$

$5x - 2x < 10 + 3$

$3x < 13$

$x < \frac{13}{3}$

3) $5x + 2y = 20$

i) On x-axis $y = 0$

$\Rightarrow 5x + 0 = 20$

$\Rightarrow x = 4$

Point is $(4, 0)$

ii) $5x + 2y = 20$ ①

$y = 5 - x$ ②

Subst for y in ①

$5x + 2(5 - x) = 20$

$5x + 10 - 2x = 20$

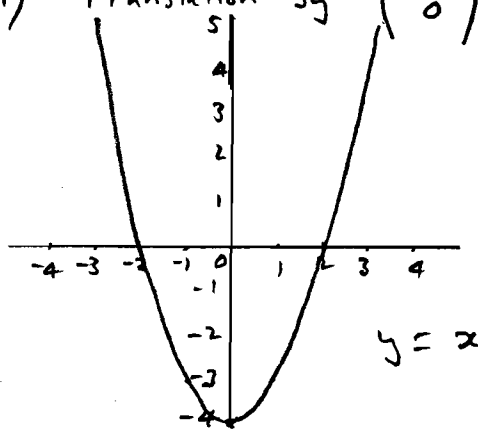
$5x - 2x = 20 - 10$

$3x = 10$

$x = \frac{10}{3}$

4) i) Translation by $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$

ii)



$y = x^2 - 4$

5) i) $144^{-\frac{1}{2}} = \frac{1}{\sqrt{144}} = \frac{1}{12}$

ii)

$\frac{1}{5+\sqrt{7}} + \frac{4}{5-\sqrt{7}}$

$= \frac{1(5-\sqrt{7}) + 4(5+\sqrt{7})}{(5+\sqrt{7})(5-\sqrt{7})}$

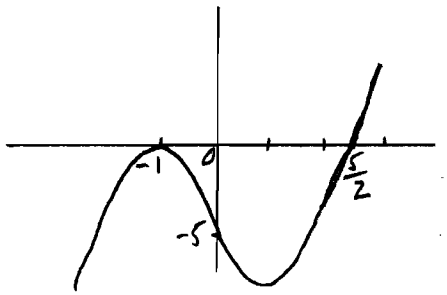
$= \frac{5 - \sqrt{7} + 20 + 4\sqrt{7}}{25 - 7}$

$= \frac{25 + 3\sqrt{7}}{18}$

6) $f(x) = (x+1)^2(2x-5)$

6 cont)

i)



ii) $f(x) = (x+1)^2(2x-5)$

$f(x) = (x^2+2x+1)(2x-5)$

$f(x) = 2x^3 + 4x^2 + 2x - 5x^2 - 10x - 5$

$f(x) = 2x^3 - x^2 - 8x - 5$

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7) Let $f(x) = x^3 + 2x^2 + 5x + k$

By remainder theorem $f(-3) = 6$

$(-3)^3 + 2(-3)^2 + 5(-3) + k = 6$

$-27 + 18 - 15 + k = 6$

$-24 + k = 6$

$k = 6 + 24$

$k = 30$

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8) $\left(x + \frac{5}{x}\right)^3$

$= x^3 + 3x^2 \times \frac{5}{x} + 3x \times \left(\frac{5}{x}\right)^2 + \left(\frac{5}{x}\right)^3$

$= x^3 + 15x + \frac{75}{x} + \frac{125}{x^3}$

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9) $\begin{cases} y = x^2 - 5x + 7 & (1) \\ y = 3x - 10 & (2) \end{cases}$

Subst for y in (1)

$3x - 10 = x^2 - 5x + 7$

$0 = x^2 - 5x + 7 - 3x + 10$

$0 = x^2 - 8x + 17$

Discriminant $b^2 - 4ac$

$= 64 - 68 = -4$

No real roots since discriminant < 0

$\therefore$ no points of intersection

Section B

10) A(-2, -1)
B(6, 3)
C(3, 5)
D(-1, 3)

i) grad AB $= \frac{3 - -1}{6 - -2} = \frac{4}{8} = \frac{1}{2}$

grad DC $= \frac{5-3}{3--1} = \frac{2}{4} = \frac{1}{2}$

Same gradient $\therefore$ parallel

ii) $|AD| = \sqrt{(-2--1)^2 + (-1-3)^2}$

$= \sqrt{1 + 16} = \sqrt{17}$

$|BC| = \sqrt{(6-3)^2 + (3-5)^2}$

$= \sqrt{9 + 4} = \sqrt{13}$

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10ii cont) $|AD| \neq |BC|$

$\therefore$ trapezium not isosceles

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iii) Eqn of BD is $y = 3$ (1)

Find eqn of AC

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y--1}{5--1} = \frac{x--2}{3--2}$$

$$\frac{y+1}{6} = \frac{x+2}{5}$$

$$5(y+1) = 6(x+2)$$

$$5y+5 = 6x+12$$

$$5y = 6x+7 \quad (2)$$

Solve (1) and (2) simultaneously to find M

Subst for y in (2)

$$15 = 6x+11$$

$$15-11 = 6x$$

$$4 = 6x$$

$$x = \frac{2}{3}$$

M is point $\left(\frac{2}{3}, 3\right)$

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iv) Midpoint of BD $= \left(\frac{6+-1}{2}, 3\right)$

$= \left(\frac{5}{2}, 3\right)$

Midpoint of AC $= \left(\frac{-2+3}{2}, \frac{-1+5}{2}\right)$

$= \left(\frac{1}{2}, 2\right)$

Neither of these points coincide with M so neither is bisected by the other.

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11) $(x-3)^2 + (y+2)^2 = 25$

i) Centre $(3, -2)$ radius $= 5$

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ii) Subst $(6, -6)$ in eqn of circle

$(6-3)^2 + (-6+2)^2$

$= 3^2 + 4^2 = 25$ ✓

$\therefore (6, -6)$ on circle

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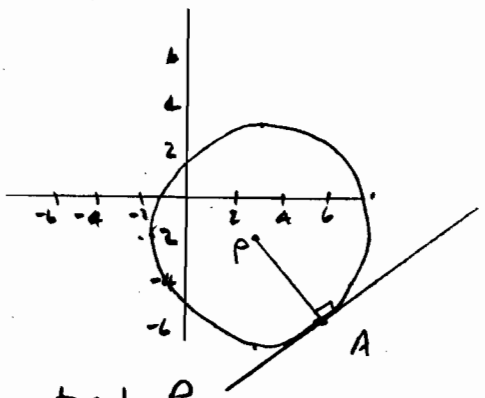
iii)



Let centre be P

Grad AP $= \frac{-6--2}{6-3} = \frac{-4}{3} = -\frac{4}{3}$

$\therefore$ grad of tgt at A $= +\frac{3}{4}$

11 iii cont) Using $y - y_1 = m(x - x_1)$

$y - -6 = \frac{3}{4}(x - 6)$

$y + 6 = \frac{3}{4}x - \frac{18}{4}$

$y = \frac{3}{4}x - \frac{9}{2} - 6$

$y = \frac{3}{4}x - \frac{21}{2}$

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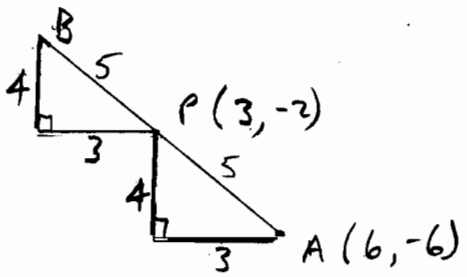
ii) Part omitted from (ii)



By congruent Δs

B is point $(3-3, -2+4)$

$= (0, 2)$

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iv) Again by congruent Δs

C is point $(6+6, -6-8)$

$= (12, -14)$

Radius 10 so circle is

$(x-12)^2 + (y+14)^2 = 100$

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12) $y = \frac{1}{5}x(10 - x)$

i) $x = 10$ m

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ii) Max occurs when $x = \frac{10+0}{2} = 5$ m

$y = \frac{5}{5}(10-5) = 5$ m

Max occurs at $(5, 5)$

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iii) By symmetry about $x = 5$

$d = 5 - 1.5 = 3.5$ m

When $x = d = \frac{7}{2}$, $y = \frac{1}{5} \times \frac{7}{2}(10 - \frac{7}{2})$

$y = \frac{7}{10} \times \frac{13}{2} = \frac{91}{20}$

$y = 4.55$ m which is higher than lorry

Arch ∴ higher than lorry across whole width of lorry so lorry can pass through arch.

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iv) If $y = 4.5$ m

$\frac{9}{2} = \frac{x}{5}(10 - x)$

$\frac{45}{2} = x(10-x) = 10x - x^2$

$45 = 20x - 2x^2$

$2x^2 - 20x + 45 = 0$

$x = \frac{20 \pm \sqrt{400 - 360}}{4}$

$x = \frac{20 \pm \sqrt{40}}{4} = \frac{20 \pm 2\sqrt{10}}{4}$

$x = \frac{10 \pm \sqrt{10}}{2}$

Width of lorry $= \frac{10+\sqrt{10}}{2} - \frac{10-\sqrt{10}}{2}$

$= \frac{2\sqrt{10}}{2} = \sqrt{10}$ m