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1) $y = 5x + 2$

$\perp$ line has gradient $-\frac{1}{5}$

Passes through $(1,6)$

using $y - y_1 = m(x - x_1)$

$y - 6 = -\frac{1}{5}(x - 1)$

$y - 6 = -\frac{1}{5}x + \frac{1}{5}$

$y = -\frac{1}{5}x + \frac{31}{5}$

4) i) $(7 + 3\sqrt{2})(5 - 2\sqrt{2})$

$= 35 + 15\sqrt{2} - 14\sqrt{2} - 6 \times 2$

$= 23 + \sqrt{2}$

ii) $\sqrt{54} + \frac{12}{\sqrt{6}}$

$= \sqrt{9 \times 6} + \frac{6 \times 2}{\sqrt{6}}$

$= 3\sqrt{6} + 2\sqrt{6} = 5\sqrt{6}$

2) i) $9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{3}$

ii) $\frac{(4x^4)^3 y^2}{2x^2 y^5}$

$= \frac{64x^{12}y^2}{2x^2y^5}$

$= \frac{32x^{10}}{y^3}$ or $32x^{10}y^{-3}$

5) $\frac{2x+1}{5} < \frac{3x+4}{6}$

$6(2x+1) < 5(3x+4)$

$12x + 6 < 15x + 20$

$12x - 15x < +20 - 6$

$-3x < 14$

$x > \frac{14}{-3}$

$x > -\frac{14}{3}$

3) $(n+2)^3 - n^3$

$= (n+2)(n^2+4n+4) - n^3$

$= n^3 + 2n^2$
$\quad + 4n^2 + 8n$
$\quad\quad + 4n + 8 - n^3$

$= 6n^2 + 12n + 8$

6) $4h + 5 = 9a - ha^2$

$4h + ha^2 = 9a - 5$

$h(4 + a^2) = 9a - 5$

$h = \frac{9a - 5}{4 + a^2}$

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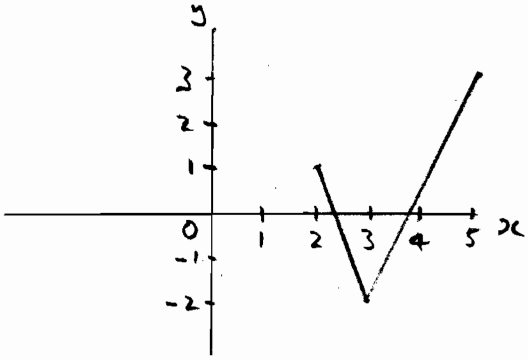
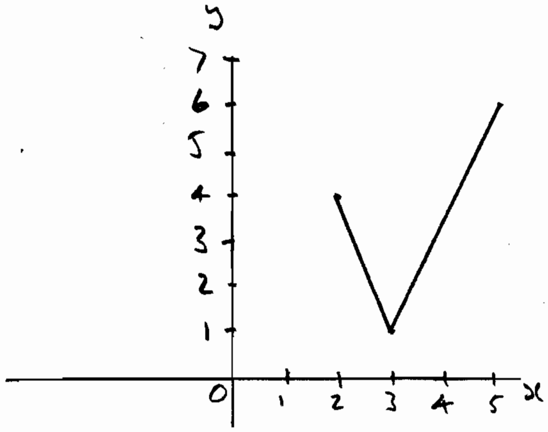
7) $y = g(x)$



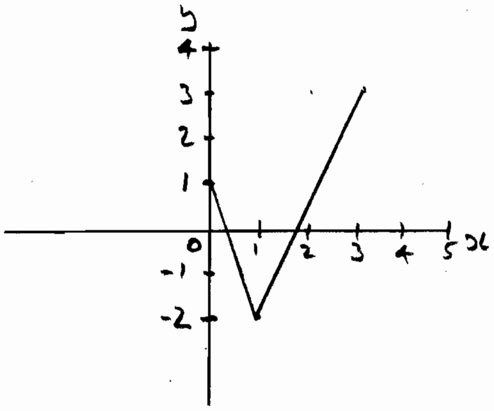
i) $y = g(x) + 3$

requires translation by $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$



ii) $y = g(x+2)$

requires translation by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$



8) $5x^2 + 15x + 12$

$= 5\left(x^2 + 3x + \frac{12}{5}\right)$

$= 5\left(\left(x + \frac{3}{2}\right)^2 + \frac{12}{5} - \frac{9}{4}\right)$

$= 5\left(x + \frac{3}{2}\right)^2 + 12 - \frac{45}{4}$

$= 5\left(x + \frac{3}{2}\right)^2 + \frac{3}{4}$

Min value of y when

$y = 5\left(x + \frac{3}{2}\right)^2 + \frac{3}{4}$

is given by $y = \frac{3}{4}$

9)

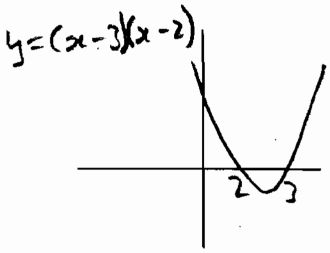
i) (n^3+1) odd $\Leftarrow$ n even

If n is even then n^3 is even so n^3+1 is odd

However, if n^3+1 is odd then n^3 is even

but $n = \sqrt[3]{n^3}$ not necessarily an integer

ii) $(x-3)(x-2) > 0 \Leftarrow x > 3$



$x > 3 \Rightarrow (x-3)(x-2) > 0$ from graph

but when $x < 2$, $(x-3)(x-2) > 0$ and $x < 3$

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10) i) $A(4,7)$
$B(2,1)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-7}{1-7} = \frac{x-4}{2-4}$$

$$\frac{y-7}{-6} = \frac{x-4}{-2}$$

$$y-7 = \frac{-6(x-4)}{-2}$$

$$y-7 = 3(x-4)$$

$$y-7 = 3x-12$$

$$y = 3x-5$$

ii) $C(-1,2)$

Gradient AB $= \dfrac{y_2-y_1}{x_2-x_1}$

$$= \frac{1-7}{2-4} = \frac{-6}{-2} = 3$$

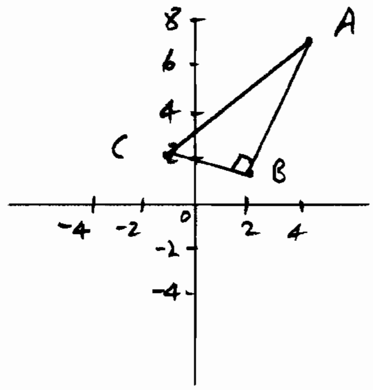
Gradient BC

$$= \frac{1-2}{2--1} = \frac{-1}{3} = -\frac{1}{3}$$

AB and BC are $\perp$

since $3 \times -\frac{1}{3} = -1$

$\therefore \angle ABC = 90°$



$$|AB| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(4-2)^2+(7-1)^2}$$

$$= \sqrt{4+36} = \sqrt{40}$$

$$|BC| = \sqrt{(2--1)^2+(1-2)^2}$$

$$= \sqrt{9+1} = \sqrt{10}$$

Area $= \frac{1}{2}$ base $\times$ height

$$= \frac{1}{2}\sqrt{40} \times \sqrt{10}$$

$$= \frac{1}{2}\sqrt{400}$$

$$= \frac{1}{2} \times 20$$

$$= 10 \text{ units}^2$$

iii) Midpoint $D = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$$D = \left(\frac{4+-1}{2}, \frac{7+2}{2}\right)$$

$$D\left(\frac{3}{2}, \frac{9}{2}\right)$$

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10iii) cont) A circle could be drawn with diameter AC, centre D

B would be on the circle because $\angle ABC = 90°$, an angle in a semi-circle

Thus A, B, C would all be points on the circle, centre D, and therefore equidistant from D

11) $f(x) = 2x^3 - 3x^2 - 23x + 12$

i) $f(-3) = 2(-3)^3 - 3(-3)^2 - 23(-3) + 12$

$= -54 - 27 + 69 + 12$

$= -81 + 81 = 0$

$\therefore x = -3$ a root of $f(x) = 0$

$\Rightarrow (x+3)$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & 2x^2 - 9x + 4 \\
\hline
x+3 & 2x^3 - 3x^2 - 23x + 12 \\
 & 2x^3 + 6x^2 \\
\hline
 & \quad -9x^2 - 23x \\
 & \quad -9x^2 - 27x \\
\hline
 & \qquad +4x + 12
\end{array}
$$

$f(x) = (x+3)(2x^2 - 9x + 4)$

Factorise $2x^2 - 9x + 4$

$2 \times 4 = 8$
-1 and -8

$2x^2 - 8x - x + 4$

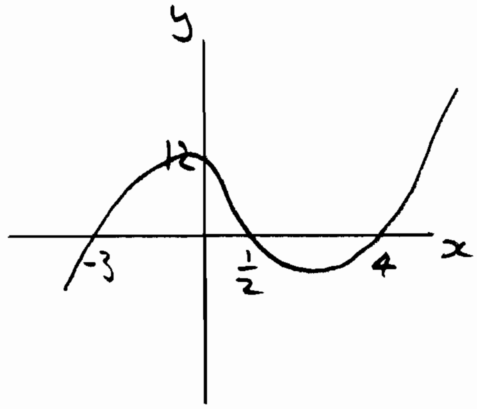
$2x(x-4) - 1(x-4)$

$(2x-1)(x-4)$

$f(x) = (x+3)(2x-1)(x-4)$

11ii y-intercept = +12

$f(x) = 0$ at $-3, \frac{1}{2}, 4$



11iii)

$y = 4x + 12$ ①

$y = 2x^3 - 3x^2 - 23x + 12$ ②

Sub for y in ②

$4x + 12 = 2x^3 - 3x^2 - 23x + 12$

$0 = 2x^3 - 3x^2 - 23x + 12 - 4x - 12$

$0 = 2x^3 - 3x^2 - 27x$

$0 = x(2x^2 - 3x - 27)$

Factorise $(2x^2 - 3x - 27)$

$2 \times -27 = -54$
$-9 \quad +6$

$2x^2 - 9x + 6x - 27$

$x(2x-9) + 3(2x-9)$

$(x+3)(2x-9)$

$\therefore \; 0 = x(x+3)(2x-9)$

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11 iii) cont)

$\Rightarrow x = 0$

$x = -3$

$x = \frac{9}{2}$

12) i) $(x-2)^2 + y^2 = 20$

Centre $(2, 0)$, radius $\sqrt{20}$

ii) On y-axis $x = 0$

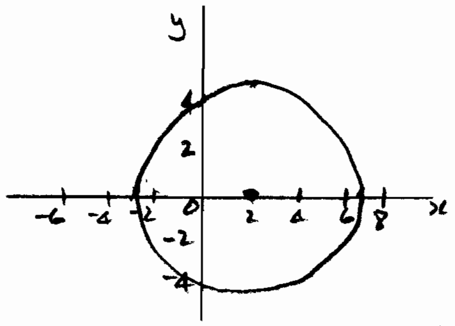
$(0-2)^2 + y^2 = 20$

$4 + y^2 = 20$

$y^2 = 16$

$y = \pm 4$

Points of intersection with y-axis are $(0,4)$ and $(0,-4)$



centre (2,0)

12) iii) $y = (2x+k)$

$(x-2)^2 + y^2 = 20$

Sub for y in circle

$(x-2)^2 + (2x+k)^2 = 20$

$x^2 - 4x + 4 + 4x^2 + 4xk + k^2 = 20$

$5x^2 + (4k-4)x + k^2 - 16 = 0$

iv) Tangent when $b^2 - 4ac = 0$

$(4k-4)^2 - 4(5)(k^2-16) = 0$

$16k^2 - 32k + 16 - 20k^2 + 320 = 0$

$-4k^2 - 32k + 336 = 0$

$4k^2 + 32k - 336 = 0$

$k^2 + 8k - 84 = 0$

$(k-6)(k+14) = 0$

$\Rightarrow k = 6$ or $k = -14$