1) Parallel to $y = 3x + 1$

$\Rightarrow$ gradient $= 3$

thru $(4, 5)$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 3(x - 4)$$

$$y - 5 = 3x - 12$$

$$y = 3x - 7$$

2)
i) $(5a^2b)^3 \times 2b^4$

$= 125a^6b^3 \times 2b^4$

$= 250a^6b^7$

ii) $\left(\frac{1}{16}\right)^{-1} = 16$

iii) $16^{3/2} = (\sqrt[2]{16})^3 = 4^3 = 64$

3)
$$a = \frac{\sqrt{y} - 5}{c}$$

$$ac = \sqrt{y} - 5$$

$$ac + 5 = \sqrt{y}$$

$$y = (ac + 5)^2$$

4)
$$2(1 - x) > 6x + 5$$

$$2 - 2x > 6x + 5$$

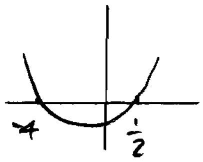
$$-2x - 6x > +5 - 2$$

$$-8x > 3$$

$$x < \frac{3}{-8}$$

$$x < -\frac{3}{8}$$

ii)
$$(2x - 1)(x + 4) < 0$$



$y = (2x-1)(x+4)$

$$-4 < x < \frac{1}{2}$$

5)
i) $\sqrt{48} + \sqrt{27}$

$= \sqrt{16 \times 3} + \sqrt{9 \times 3}$

$= 4\sqrt{3} + 3\sqrt{3} = 7\sqrt{3}$

ii)
$$\frac{5\sqrt{2}}{3 - \sqrt{2}} = \frac{5\sqrt{2}}{3 - \sqrt{2}} \times \frac{3 + \sqrt{2}}{3 + \sqrt{2}}$$

$$= \frac{15\sqrt{2} + 10}{9 - 2}$$

$$= \frac{10 + 15\sqrt{2}}{7}$$

6) $5 + 2k = 29$

$\Rightarrow 2k = 24 \Rightarrow k = 12$

Using remainder theorem with $f(3)$

$$3^3 + 12(3) + m = 59$$

$$27 + 36 + m = 59$$

6) cont)

$m = 59 - 63$

$m = -4$

Solution $k = 12$, $m = -4$

7)

$\left(1 + \frac{1}{2}x\right)^4$

1 2 1
1 3 3 1
1 4 6 4 1

$1 + 4\left(\frac{1}{2}\right)x + 6\left(\frac{1}{2}x\right)^2 + 4\left(\frac{1}{2}x\right)^3 + \left(\frac{1}{2}x\right)^4$

$= 1 + 2x + \frac{3x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16}$

8)

$5x^2 + 20x + 6$

$= 5\left[x^2 + 4x + \frac{6}{5}\right]$

$= 5\left[(x+2)^2 + \frac{6}{5} - 4\right]$

$= 5(x+2)^2 + 6 - 20$

$= 5(x+2)^2 - 14$

9)

$x - 5 = 0 \Leftrightarrow x^2 = 25$

not true since when $x = -5$

$x^2 = 25$

but $x - 5 = -5 - 5 = -10$

10) i)

$2x^2 - x - 3 = 0$

$(2x-3)(x+1) = 0$

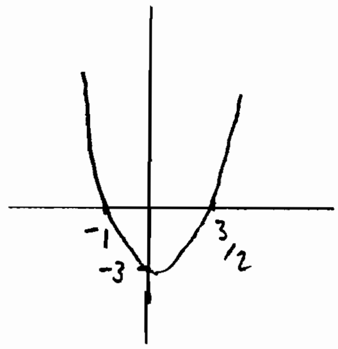
Either $2x - 3 = 0$

$\Rightarrow x = \frac{3}{2}$

or $x + 1 = 0$

$\Rightarrow x = -1$

ii)



iii)

$x^2 - 5x + 10 = 0$

Discriminant $b^2 - 4ac$

$= 25 - 40 = -15$

Discriminant < 0 $\therefore$ no real roots

iv)

$y = 2x^2 - x - 3$ (1)

$y = x^2 - 5x + 10$ (2)

Subst for y in (2)

$2x^2 - x - 3 = x^2 - 5x + 10$

$x^2 + 4x - 13 = 0$

$x = \frac{-4 \pm \sqrt{16 + 52}}{2}$

$x = \frac{-4 \pm \sqrt{68}}{2} = \frac{-4 \pm 2\sqrt{17}}{2}$

$x = -2 \pm \sqrt{17}$

11) $A(-1,3)$ $B(5,1)$

i)
$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$
$$\frac{y-3}{1-3} = \frac{x--1}{5--1}$$
$$\frac{y-3}{-2} = \frac{x+1}{6}$$
$$6(y-3) = -2(x+1)$$
$$y-3 = -\frac{1}{3}(x+1)$$
$$y-3 = -\frac{1}{3}x - \frac{1}{3}$$
$$y = -\frac{1}{3}x + \frac{8}{3}$$

ii) Cuts y-axis at $\frac{8}{3}$

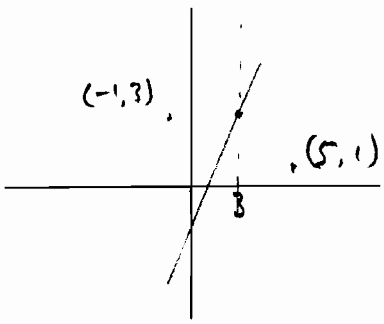
Cuts x-axis when
$$0 = -\frac{1}{3}x + \frac{8}{3}$$
$$\frac{1}{3}x = \frac{8}{3} \Rightarrow x = 3$$

Area $= \frac{1}{2}$ base $\times$ height
$$= \frac{1}{2} \times 8 \times \frac{8}{3}$$
$$= \frac{64}{6} = \frac{32}{3} \text{ units}^2$$

iii) Midpoint of AB $= \left(\frac{-1+5}{2}, \frac{3+1}{2}\right)$
$$= (2, 2)$$

Gradient of $\perp$ bisector $= +3$
$$y - y_1 = m(x-x_1)$$
$$y-2 = 3(x-2)$$
$$y-2 = 3x-6$$
$$y = 3x-4$$

iv)



Centre lies on $\perp$ bisector of AB with $x=3$, so $y = 3(3)-4$, $y = 5$

Centre $(3,5)$

$$\text{radius} = \sqrt{(3-5)^2 + (5-1)^2} = \sqrt{4+16} = \sqrt{20}$$

Eqn of circle
$$(x-3)^2 + (y-5)^2 = 20$$

12) $f(x) = x^3 + 6x^2 - x - 30$

i) $f(3) = 27 + 6(9) - 3 - 30 \neq 0$

$f(2) = 8 + 24 - 2 - 30 = 0$ ✓

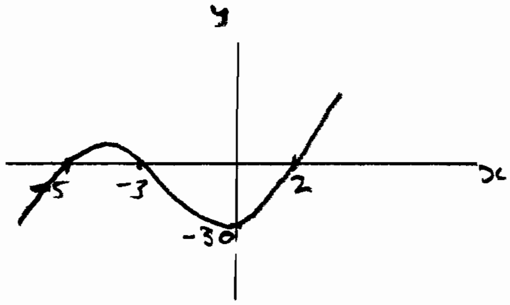
$\therefore (x-2)$ is a factor

$$
\begin{array}{r|l}
 & x^2 + 8x + 15 \\
x-2 & x^3 + 6x^2 - x - 30 \\
 & \underline{x^3 - 2x^2} \\
 & 8x^2 - x \\
 & \underline{8x^2 - 16x} \\
 & +15x - 30 \\
 & \underline{+15x - 30}
\end{array}
$$

$f(x) = (x-2)(x^2 + 8x + 15)$

$f(x) = (x-2)(x+5)(x+3)$

ii)



iii) Find $f(x-1)$

$= (x-1)^3 + 6(x-1)^2 - (x-1) - 30$

$= (x^3 - 3x^2 + 3x - 1)$

$+ 6(x^2 - 2x + 1) - x + 1 - 30$

$= x^3 - 3x^2 + 3x - 1$
$\quad + 6x^2 - 12x + 6$
$\quad - x - 29$

$= x^3 + 3x^2 - 10x - 24$