1) $n^2 + n = n(n+1)$

This is either odd × even = even
or even × odd = even

for any positive integer n

∴ result is always even

2) i) translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

ii) $y = f(x-2)$

3) $(2+x)^4$

$$\begin{matrix} & & 1 & 2 & 1 & & \\ & 1 & 3 & 3 & 1 & & \\ 1 & 4 & 6 & 4 & 1 & & \end{matrix}$$

$= 2^4 + 4(2)^3 x + 6(2)^2 x^2$
$\quad + 4(2)x^3 + x^4$

$= 16 + 32x + 24x^2 + 8x^3 + x^4$

4) $\dfrac{3(2x+1)}{4} > -6$

$3(2x+1) > -24$

$2x+1 > -8$

$2x > -8-1$

$2x > -9$

$x > -\dfrac{9}{2}$

5) $P = \dfrac{C}{C+4}$

$(C+4)P = C$

$CP + 4P = C$

$4P = C - CP$

$4P = C(1-P)$

$C = \dfrac{4P}{1-P}$

6) Let $f(x) = x^3 + 3x + k$

By remainder theorem

$f(1) = 6$

$1^3 + 3\times 1 + k = 6$

$1 + 3 + k = 6$

$\Rightarrow \quad k = 2$

7) Gradient of AB $= 4$

∴ gradient of BC $= -\frac{1}{4}$

Using $\quad y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{4}(x-2)$

$y - 3 = -\frac{1}{4}x + \frac{1}{2}$

$y = -\frac{1}{4}x + \frac{7}{2}$

is line ⊥ to AB passing thro $(2,3)$

When $y=0$, $\quad 0 = -\frac{1}{4}x + \frac{7}{2}$
$\quad 0 = -x + 14$

$\Rightarrow x = 14 \quad$ x coord of $C = 14$

8) i)

$$5\sqrt{8} + 4\sqrt{50}$$
$$= 5\sqrt{2\times4} + 4\sqrt{25\times2}$$
$$= 10\sqrt{2} + 20\sqrt{2}$$
$$= 30\sqrt{2}$$

ii)

$$\frac{\sqrt{3}}{6-\sqrt{3}}$$
$$= \frac{\sqrt{3}}{6-\sqrt{3}} \times \frac{(6+\sqrt{3})}{(6+\sqrt{3})}$$
$$= \frac{6\sqrt{3}+3}{36-3}$$
$$= \frac{6\sqrt{3}}{33} + \frac{3}{33}$$
$$= \frac{2\sqrt{3}}{11} + \frac{1}{11}$$

9) i)

$$x^2 + 5x + k = 0$$

For real roots $b^2 - 4ac \geq 0$

$$25 - 4k \geq 0$$
$$25 \geq 4k$$
$$k \leq \frac{25}{4}$$

ii)

$$4x^2 + 20x + 25 = 0$$
$$(2x+5)(2x+5) = 0$$
$$\Rightarrow x = -\frac{5}{2}$$

10) $x^2 + y^2 = 45$

i) Centre $(0,0)$

Radius $= \sqrt{45}$

ii)

$$x^2 + y^2 = 45 \quad (1)$$
$$x + y = 3 \quad (2)$$

from (2) $y = 3 - x$

Subst for y in (1)

$$x^2 + (3-x)^2 = 45$$
$$x^2 + 9 - 6x + x^2 = 45$$
$$2x^2 - 6x - 36 = 0$$
$$x^2 - 3x - 18 = 0$$
$$(x-6)(x+3) = 0$$
$$\Rightarrow x = 6 \text{ or } x = -3$$

when $x = 6$, $y = -3$ when $x = -3$, $y = 6$

A and B are $(6,-3)$ and $(-3,6)$

$$|AB| = \sqrt{(6--3)^2 + (-3-6)^2}$$
$$= \sqrt{9^2 + 9^2}$$
$$= \sqrt{162}$$

11) i)

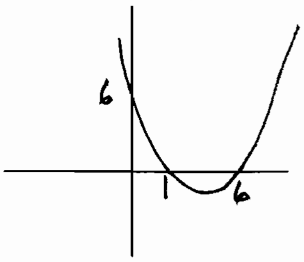
$$x^2 - 7x + 6$$
$$= \left(x - \frac{7}{2}\right)^2 + 6 - \frac{49}{4}$$
$$= \left(x - \frac{7}{2}\right)^2 - \frac{25}{4}$$

ii) Min point $\left(\frac{7}{2}, -\frac{25}{4}\right)$

iii) When $x^2 - 7x + 6 = 0$
$(x-6)(x-1) = 0$
$\Rightarrow x = 6$ or $x = 1$

Crosses x axis at $(1,0)$ and $(6,0)$

Crosses y axis at $(0,6)$



iv)

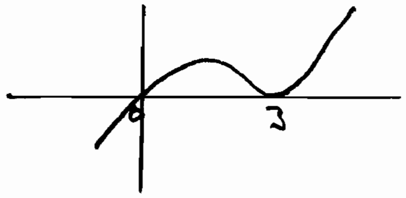
$$y = x^2 - 7x + 6$$
$$y = x^2 - 3x + 4$$

Intersect when

$$\cancel{x^2} - 7x + 6 = \cancel{x^2} - 3x + 4$$
$$\Rightarrow 6 - 4 = -3x + 7x$$
$$\Rightarrow 2 = 4x$$
$$\Rightarrow x = \frac{1}{2}$$

$\therefore$ only one point of intersection when $x = \frac{1}{2}$

12) i) $y = x(x-3)^2$



ii)

$$x(x-3)^2 = 2$$
$$\Rightarrow x(x^2 - 6x + 9) = 2$$
$$\Rightarrow x^3 - 6x^2 + 9x = 2$$
$$\Rightarrow x^3 - 6x^2 + 9x - 2 = 0$$

iii)

$$2^3 - 6 \times 2^2 + 9 \times 2 - 2$$
$$= 8 - 24 + 18 - 2 = 0$$

$\therefore x = 2$ is a root.

$$\begin{array}{r|l} & x^2 - 4x + 1 \\ \hline x - 2 & x^3 - 6x^2 + 9x - 2 \\ & \underline{x^3 - 2x^2} \\ & \quad -4x^2 + 9x \\ & \quad \underline{-4x^2 + 8x} \\ & \qquad\quad x - 2 \end{array}$$

$$(x-2)(x^2 - 4x + 1) = 0$$
$$x = \frac{4 \pm \sqrt{16 - 4}}{2}$$
$$x = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$$
$$x = 2 + \sqrt{3} \text{ or } x = 2 - \sqrt{3}$$