1) Line parallel to $y = 2x + 7$ is of form

$$y = 2x + c$$

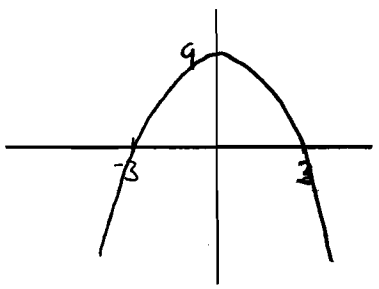
$(3, 10)$ on line

$$\Rightarrow \quad 10 = 2 \times 3 + c$$

$$\Rightarrow \quad c = 4$$

Line is $y = 2x + 4$

2)



Passes through $(-3, 0)$
$(3, 0)$
$(0, 9)$

3)

$$2a + 5c = af + 7c$$

$$2a - af = 7c - 5c$$

$$a(2 - f) = 2c$$

$$a = \frac{2c}{2 - f}$$

4)

Let $f(x) = x^3 + kx + 5$

If division by $(x-2)$ gives a remainder of 3 then by remainder theorem

$$f(2) = 3$$

$$2^3 + 2k + 5 = 3$$

$$8 + 2k + 5 = 3$$

$$2k = 3 - 13$$

$$2k = -10$$

$$k = -5$$

5)

$$(x + 5)^6$$

Term in x^4

$$= {}^6C_4 \, x^4 (5)^2$$

$$= \frac{6 \times 5}{2 \times 1} \times 25 x^4$$

$$= 375 x^4$$

Coefficient $= 375$

6)

i) $25^{3/2} = \left(\sqrt{25}\right)^3$

$= 5^3$

$= 125$

ii) $\left(\frac{7}{3}\right)^{-2} = \frac{1}{\left(\frac{7}{3}\right)^2}$

$= \frac{1}{\left(\frac{49}{9}\right)}$

$= \frac{9}{49}$

7) $a = \frac{3}{2}$, $b = \frac{9-\sqrt{17}}{4}$

$c = \frac{9+\sqrt{17}}{4}$

$a+b+c =$

$\frac{6}{4} + \frac{9-\sqrt{17}}{4} + \frac{9+\sqrt{17}}{4}$

$= \frac{6+9-\sqrt{17}+9+\sqrt{17}}{4}$

$= \frac{24}{4} = 6$

$abc =$

$\frac{3}{2}\left(\frac{9+\sqrt{17}}{4}\right)\left(\frac{9-\sqrt{17}}{4}\right)$

$= \frac{3\left(9^2-\sqrt{17}^2\right)}{2\times4\times4}$

$= \frac{3(81-17)}{32}$

$= \frac{3\times 64^2}{32_1} = 6$

$\therefore a+b+c = abc$

8)

$2x^2 + kx + 2 = 0$

If no real roots $b^2 < 4ac$

$k^2 < 4\times2\times2$

$k^2 < 16$

$\Rightarrow -4 < k < 4$

9) i) $3a^3b \times 4(ab)^2$

$= 3a^3b \times 4a^2b^2$

$= 12a^5b^3$

ii) $x^2-4 = (x+2)(x-2)$

$x^2-5x+6 = (x-2)(x-3)$

$\frac{x^2-4}{x^2-5x+6} = \frac{(x+2)(x-2)}{(x-2)(x-3)}$

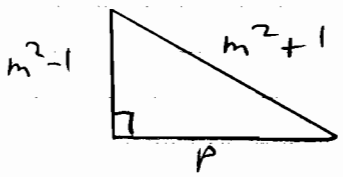
$= \frac{x+2}{x-3}$

10) $(m^2+1)^2 - (m^2-1)^2$

Difference of two squares

$= (m^2+1+(m^2-1))(m^2+1-(m^2-1))$

$= (2m^2)(2) = 4m^2$



By Pythagoras $p^2 + (m^2-1)^2 = (m^2+1)^2$

$p^2 = (m^2+1)^2 - (m^2-1)^2$

$p^2 = 4m^2$

$p = 2m$

11) i) From graph

A) $x = 3.8$ and $x = 0.25$

B) $2x + \frac{1}{x} = 4$

$x + \frac{1}{x} = 4 - x$

Draw graph of $y = 4 - x$ and read x coords of intersection with $y = x + \frac{1}{x}$

$x = 0.3$ and $x = 1.7$

11 ii) $(x-1)^2 + y^2 = 4$

On y-axis $x = 0$

$(0-1)^2 + y^2 = 4$

$1 + y^2 = 4$

$\Rightarrow y^2 = 3, \quad y = \pm\sqrt{3}$

Coords $(0, \sqrt{3})$ and $(0, -\sqrt{3})$

11 iii) Centre $(1, 0)$, radius 2

Point $(1, 2)$ is on circle at highest point and lowest point of right branch of $y = x + \frac{1}{x}$

$\therefore$ circle touches graph

Lowest point of circle is $(1, -2)$ For all other x values y coord > -2 so circle does not intersect left branch of graph which has highest point at $(-1, -2)$

12) A $(-1, 4)$
B $(7, 8)$

i) $M = \left(\frac{-1+7}{2} \quad \frac{4+8}{2}\right)$

$M = (3, 6)$

Gradient of AB $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{8-4}{7--1} = \frac{4}{8} = \frac{1}{2}$

Gradient of $\perp$ bisector $= -2$

Using $y - y_1 = m(x - x_1)$

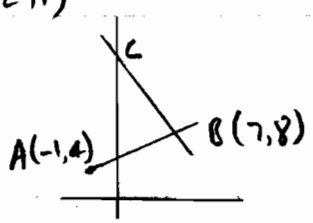
$y - 6 = -2(x - 3)$

$y - 6 = -2x + 6$

$y = -2x + 12$

$y + 2x = 12$

12 ii)



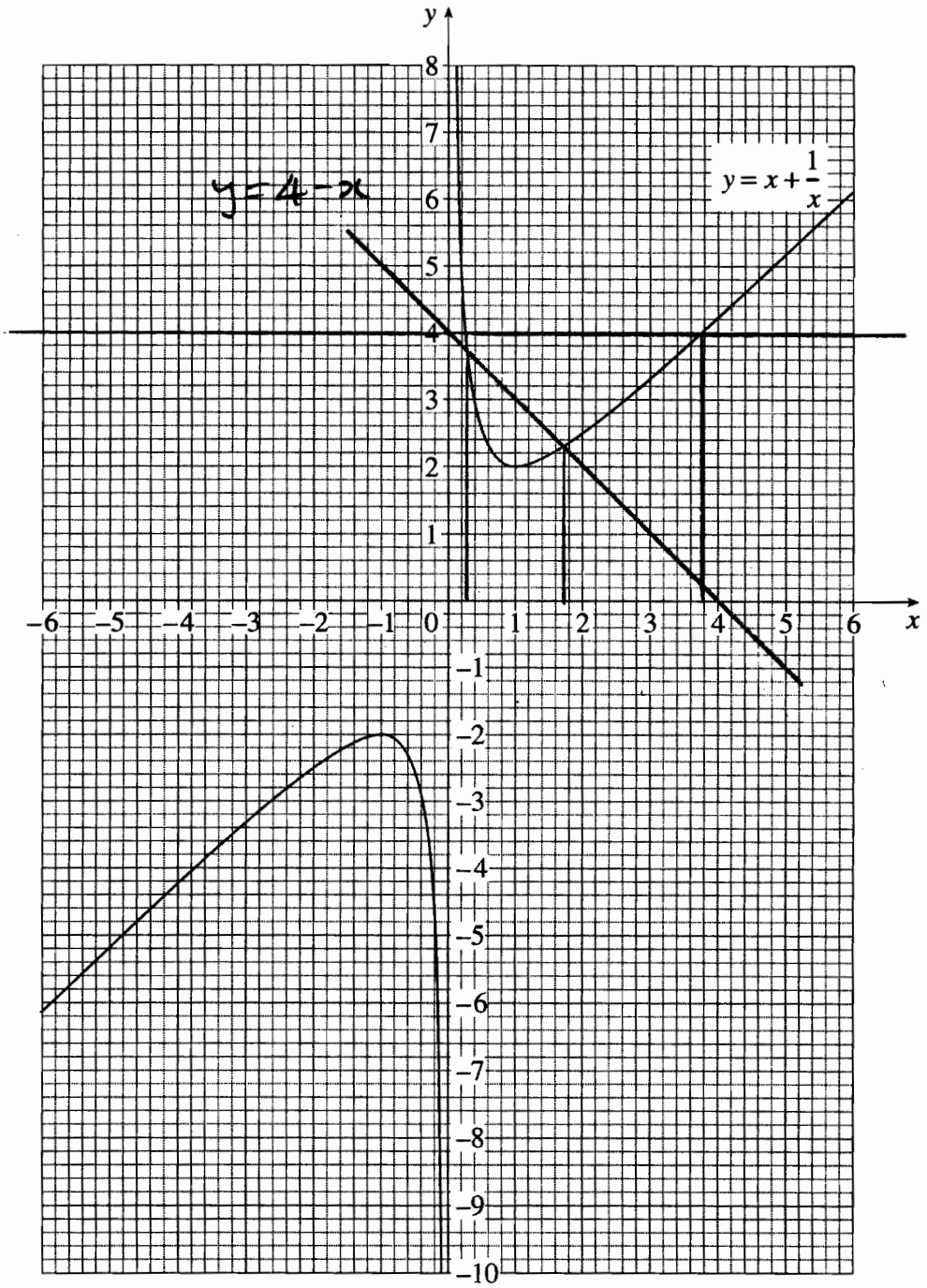
At C $x = 0$ $\quad y + 2 \times 0 = 12$

$y = 12$

$C = (0, 12)$

Find eqn of AB

11 (i)



12 ii cont)

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-4}{8-4} = \frac{x--1}{7--1}$$

$$\frac{y-4}{4} = \frac{x+1}{8}$$

$$8(y-4) = 4(x+1)$$

$$2(y-4) = x+1$$

$$y-4 = \tfrac{1}{2}x + \tfrac{1}{2}$$

$$y = \tfrac{1}{2}x + \tfrac{9}{2}$$

At D on this line $x = 0$

$\therefore y = \frac{9}{2}$

$D = (0, \frac{9}{2})$

Area of $\triangle$ = $\frac{1}{2}$ base × height

Base = CD = $12 - \frac{9}{2} = \frac{15}{2}$

height = x coord of M = 3

Area = $\frac{1}{2} \times \frac{15}{2} \times 3$

$= \frac{45}{4} = 11\frac{1}{4}$ units^2

13) i)

$f(x) = x^3 - 5x + 2$

$$\begin{array}{r} x^2+2x-1 \\ x-2 \overline{)\, x^3 \quad -5x+2} \\ x^3-2x^2 \\ \hline 2x^2-5x \\ 2x^2-4x \\ \hline -x+2 \\ -x+2 \\ \hline \end{array}$$

$f(x) = (x-2)(x^2+2x-1)$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4+4}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

Roots $x = 2$, $x = -1 \pm \sqrt{2}$

13 ii)

$f(x-3) = (x-3)^3 - 5(x-3) + 2$

$= (x^2-6x+9)(x-3) - 5(x-3) + 2$

$= x^3 - 6x^2 + 9x - 3x^2 + 18x - 27$
$\quad -5x + 15 + 2$

$= x^3 - 9x^2 + 22x - 10$

13 iii)

$f(x-3) = 0$

$f(x-3)$ obtained by translating $f(x)$ by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

Roots $x = 5$

$x = 2 + \sqrt{2}$

$x = 2 - \sqrt{2}$

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