1)

$$E = \frac{1}{2}mv^2$$

$$2E = mv^2$$

$$v^2 = \frac{2E}{m}$$

$$v = \pm\sqrt{\frac{2E}{m}}$$

2)

$$\frac{3x^2 - 7x + 4}{x^2 - 1}$$

$$= \frac{(3x-4)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$$

$$= \frac{3x-4}{x+1}$$

3)

i) $\left(\frac{1}{4}\right)^0 = 1$

ii) $16^{-\frac{3}{2}} = \frac{1}{16^{3/2}}$

$$= \frac{1}{(\sqrt[2]{16})^3}$$

$$= \frac{1}{4^3} = \frac{1}{64}$$

4)

$$y = 2x - 5 \quad (1)$$

$$6x + 2y = 7 \quad (2)$$

Subst for y in (2)

$$6x + 2(2x-5) = 7$$

$$6x + 4x - 10 = 7$$

$$10x = 17$$

$$x = 1.7$$

Subst for x in (1)

$$y = 2 \times 1.7 - 5$$

$$y = 3.4 - 5 = -1.6$$

Solution $x = 1.7,\ y = -1.6$

5)

i) $4x + 5y = 24$

$$5y = -4x + 24$$

$$y = -\frac{4}{5}x + \frac{24}{5}$$

Gradient $= -\frac{4}{5}$

ii) Line is of form $4x + 5y = c$

$(0, 12)$ on line so

$$4 \times 0 + 5 \times 12 = c$$

$$c = 60$$

Line is $4x + 5y = 60$

Cuts x axis when $y = 0$

$$\Rightarrow 4x = 60$$

$$x = 15$$

Intersection with x axis at

$$(15, 0)$$

6) Let $f(x) = x^3 + kx + 7$

Remainder when divided by $(x-2)$

$$f(2) = 2^3 + 2k + 7 = 3$$

$$8 + 2k + 7 = 3$$

$$2k = 3 - 15$$

$$2k = -12$$

$$k = -6$$

7) i

$$8C_3 = \frac{8!}{3!\,5!}$$

$$= \frac{8 \times 7 \times \cancel{6}}{\cancel{3} \times \cancel{2} \times 1} = 56$$

7ii)

$$\left(1 - \tfrac{1}{2}x\right)^8$$

Term in x^3 given by

$$8C_3\,(1)^5\left(-\tfrac{1}{2}x\right)^3$$

$$= 56 \times 1 \times -\frac{1}{8}x^3$$

$$= -7x^3$$

Coefficient $= -7$

8) i)

$$\sqrt{48} + \sqrt{3}$$

$$= \sqrt{16 \times 3} + \sqrt{3}$$

$$= 4\sqrt{3} + \sqrt{3} = 5\sqrt{3}$$

8ii)

$$\frac{1}{5+\sqrt{2}} + \frac{1}{5-\sqrt{2}}$$

$$= \frac{(5-\sqrt{2}) + (5+\sqrt{2})}{(5+\sqrt{2})(5-\sqrt{2})}$$

$$= \frac{10}{25-2} = \frac{10}{23}$$

9) i)

$3n^2 + 6n$ for even integers

we can write $n = 2m$ where m is an integer

Then

$$3n^2 + 6n = 3(2m)^2 + 6(2m)$$

$$= 12m^2 + 12m$$

$$= 12(m^2 + m)$$

$\therefore$ 12 is a factor since $(m^2 + m)$ is an integer

9ii)

Take $n = 1$

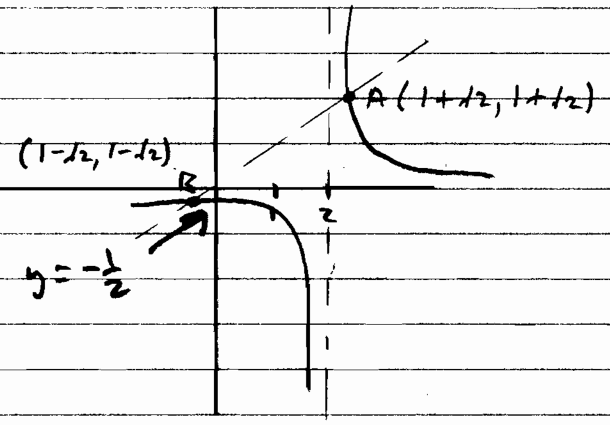
$$3n^2 + 6n = 3 + 6 = 9$$

and 12 is not a factor

$\therefore$ 12 is not a factor for all positive integers n

10)

i)



When $x = 0$ $y = \frac{1}{0-2} = -\frac{1}{2}$

ii)

$$\frac{1}{x-2} = 5$$

$$\Rightarrow \quad 1 = 5(x-2)$$

$$1 = 5x - 10$$

$$11 = 5x$$

$$x = \frac{11}{5} = 2.2$$

iii)

$$y = x$$

$$y = \frac{1}{x-2}$$

Intersect when

$$x = \frac{1}{x-2}$$

$$x(x-2) = 1$$

$$x^2 - 2x - 1 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-1)}}{2}$$

$$x = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = 1 \pm \sqrt{2}$$

On line $y = x$, so $y = 1 \pm \sqrt{2}$

Intersections at

$$(1+\sqrt{2}, 1+\sqrt{2}) \text{ and } (1-\sqrt{2}, 1-\sqrt{2})$$

11)

i)

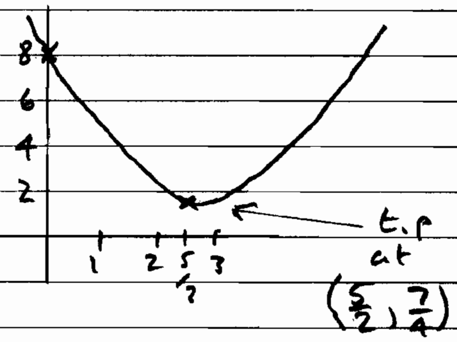
$$x^2 - 5x + 8 = (x - \tfrac{5}{2})^2 + 8 - \tfrac{25}{4}$$

$$= (x - \tfrac{5}{2})^2 + \tfrac{7}{4}$$

Min value $= \frac{7}{4}$

$\therefore$ $x^2 - 5x + 8 > 0$ for all x

ii)



iii)

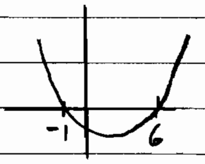
$$x^2 - 5x + 8 > 14$$

$$\Rightarrow \quad x^2 - 5x - 6 > 0$$

$$(x-6)(x+1) > 0$$

Either $x > 6$

or $x < -1$



11 iv) $f(x) = x^2 - 5x + 8$

$f(x) - 10 = x^2 - 5x + 8 - 10$

$= x^2 - 5x - 2$

Does $x^2 - 5x - 2$ have roots?

Discriminant $= b^2 - 4ac$
$= (-5)^2 - 4 \times 1 \times (-2)$
$= 25 + 8$
$= 33$

Discriminant > 0 $\therefore$ real roots

and graph crosses x axis

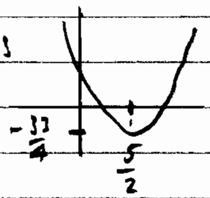
11 iv) Alternatively, from (i)

Min y for $y = x^2 - 5x + 8$ was $\frac{7}{4}$

$\therefore$ Min y for $f(x) - 10$

$= \frac{7}{4} - 10 = -\frac{33}{4}$

Below x axis



so crosses x axis

12) $x^2 + y^2 - 8x - 4y = 9$

i) $(x-4)^2 + (y-2)^2 = 9 + 16 + 4$

$(x-4)^2 + (y-2)^2 = 29$

Centre $(4, 2)$ radius $\sqrt{29}$

ii) Distance of $(0,0)$ from $(4,2)$

$= \sqrt{(4-0)^2 + (2-0)^2}$

$= \sqrt{16+4} = \sqrt{20} < \sqrt{29}$

Origin inside circle since less than a radius length from the centre

iii) $A(2,7)$ $B(6,-3)$

Show both points on circle

$(2-4)^2 + (7-2)^2 = 4 + 25 = 29$ ✓

$(6-4)^2 + (-3-2)^2 = 4 + 25 = 29$ ✓

$\therefore$ both on circle

Distance $AB = \sqrt{(2-6)^2 + (7--3)^2}$

$= \sqrt{16 + 100} = \sqrt{116}$

$= \sqrt{4 \times 29} = 2\sqrt{29}$

Since $2\sqrt{29} = 2 \times$ radius
AB is a diameter

iv) Grad from centre to A $= \frac{7-2}{2-4} = \frac{5}{-2}$

$\therefore$ gradient of tgt $= +\frac{2}{5}$

$y - y_1 = m(x - x_1)$

$y - 7 = \frac{2}{5}(x - 2)$

$y - 7 = \frac{2}{5}x - \frac{4}{5}$

$y = \frac{2}{5}x - \frac{4}{5} + 7$

$y = \frac{2}{5}x + \frac{31}{5}$