MEI CORE 1 JAN 2009

1) i) $2^{-3} = \frac{1}{8}$

ii) $9^0 = 1$

2) $(-1,-9)$ $(3,11)$

Using $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$\frac{y--9}{11--9} = \frac{x--1}{3--1}$

$\frac{y+9}{20} = \frac{x+1}{4}$

$y+9 = 5(x+1)$

$y+9 = 5x+5$

$y = 5x-4$

3) $7-x < 5x-2$

$7+2 < 5x+x$

$9 < 6x$

$1.5 < x$

Answer $x > 1.5$

4) $f(x) = x^4 + ax - 6$

$(x-2)$ a factor $\Rightarrow f(2) = 0$

$\Rightarrow 2^4 + 2a - 6 = 0$

$16 + 2a - 6 = 0$

$a = -5$

5) i) $(x^2-3)(x^3+7x+1)$

$= x^5 - 3x^3 + 7x^3 - 21x + x^2 - 3$

Coeff of x^3 is $-3+7 = +4$

ii) Term in x^2 in $(1+2x)^7$

given by ${}^7C_2 (1)^5 (2x)^2$

$= \frac{7\times6}{2\times1} \times 1 \times 4x^2$

$= 84x^2$

Coefficient of x^2 is $+84$

6) $\frac{3x+1}{2x} = 4$

$3x+1 = 8x$

$1 = 5x$

$x = \frac{1}{5}$

7) i) $125\sqrt{5} = 5^3 \times 5^{\frac{1}{2}} = 5^{7/2}$

ii) $(4a^3b^5)^2 = 16a^6b^{10}$

8) $2x^2 + kx + 18 = 0$

No real roots when $b^2 < 4ac$

$k^2 < 4\times2\times18$

$k^2 < 144$

$-12 < k < 12$

9) $y+5 = x(y+2)$

$y+5 = xy + 2x$

$y - xy = 2x - 5$

$y(1-x) = 2x-5$

$y = \dfrac{2x-5}{1-x}$

10) i) $\sqrt{75} + \sqrt{48}$

$= \sqrt{25\times3} + \sqrt{16\times3}$

$= 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$

ii) $\dfrac{14}{3-\sqrt{2}} = \dfrac{14}{3-\sqrt{2}} \times \dfrac{3+\sqrt{2}}{3+\sqrt{2}}$

$= \dfrac{42+14\sqrt{2}}{3^2-2}$

$= \dfrac{42+14\sqrt{2}}{7}$

$= 6 + 2\sqrt{2}$

Section B

11) i) Midpoint of $A(-1,0)$ and $B(11,4)$

$= \left(\dfrac{-1+11}{2}, \dfrac{0+4}{2}\right)$

$= (5, 2)$

This is centre of circle

Distance to B

$= \sqrt{(11-5)^2 + (4-2)^2} = \sqrt{36+4}$

$= \sqrt{40}$

$\therefore$ radius $= \sqrt{40}$

Eqn of circle

$(x-5)^2 + (y-2)^2 = \sqrt{40}^2$

$(x-5)^2 + (y-2)^2 = 40$

11 ii) On y axis $x = 0$

$(0-5)^2 + (y-2)^2 = 40$

$25 + (y-2)^2 = 40$

$(y-2)^2 = 15$

$y - 2 = \pm\sqrt{15}$

$y = 2 \pm \sqrt{15}$

11 iii) Gradient of AB $= \dfrac{4-0}{11--1} = \dfrac{4}{12} = \dfrac{1}{3}$

$\therefore$ Gradient of tgt at B $= -3$

$y - y_1 = m(x - x_1)$

$y - 4 = -3(x - 11)$

$y - 4 = -3x + 33$

$y = -3x + 37$

when $x = 0$, $y = 37$

when $y = 0$ $x = \dfrac{37}{3}$

Cuts axes at $(0, 37)$

and $\left(\dfrac{37}{3}, 0\right)$

12) i)

$$y = 3x^2 + 6x + 10 \quad (1)$$

$$y = 2 - 4x \quad (2)$$

Subst for y in (1)

$$2 - 4x = 3x^2 + 6x + 10$$

$$3x^2 + 10x + 8 = 0$$

$$(3x+4)(x+2) = 0$$

$$\Rightarrow x = -\frac{4}{3} \text{ or } x = -2$$

When $x = -\frac{4}{3}$ $y = 2 + \frac{16}{3} = \frac{22}{3}$

When $x = -2$ $y = 2 + 8 = 10$

Points of intersection

$$\left(-\frac{4}{3}, \frac{22}{3}\right) \text{ and } (-2, 10)$$

12 ii)

$$3x^2 + 6x + 10$$

$$= 3\left(x^2 + 2x + \frac{10}{3}\right)$$

$$= 3\left((x+1)^2 + \frac{10}{3} - 1\right)$$

$$= 3\left((x+1)^2 + \frac{7}{3}\right)$$

$$= 3(x+1)^2 + 7$$

12 iii)

Since $3(x+1)^2 \geqslant 0$ for all x

Min value of $3x^2 + 6x + 10$

is $+7$

$\therefore$ always above x-axis

13 (i) on insert

13 ii)

$$y = \frac{1}{x} \quad (1)$$

$$y = x^2 - 5x + 5 \quad (2)$$

Subst for y in (2)

$$\frac{1}{x} = x^2 - 5x + 5$$

$$\Rightarrow 1 = x^3 - 5x^2 + 5x$$

$$\Rightarrow x^3 - 5x^2 + 5x - 1 = 0 \quad (*)$$

13 iii)

$$\begin{array}{r|l} & x^2 - 4x + 1 \\ \hline x-1 & x^3 - 5x^2 + 5x - 1 \\ & x^3 - x^2 \\ \hline & \quad -4x^2 + 5x \\ & \quad -4x^2 + 4x \\ \hline & \qquad\quad x - 1 \\ & \qquad\quad x - 1 \\ \hline \end{array}$$

$$x^3 - 5x^2 + 5x - 1 = (x-1)(x^2 - 4x + 1)$$

roots of (*) are

$$x = 1$$

$$x = \frac{4 \pm \sqrt{16 - 4}}{2} = \frac{4 \pm \sqrt{12}}{2}$$

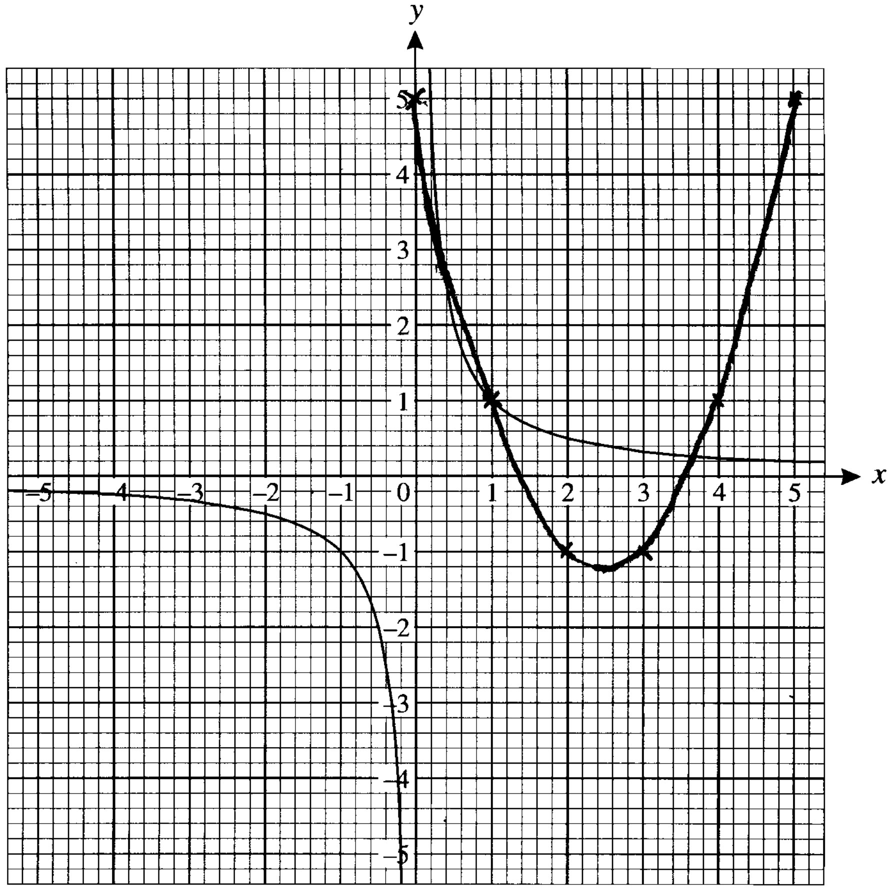
$$x = \frac{4 \pm 2\sqrt{3}}{2}$$

$$x = 2 \pm \sqrt{3}$$

$\therefore$ only one rational root, $x = 1$

4

13 (i)



x	$x^2 - 5x + 5$
0	5
1	$1 - 5 + 5 = 1$
2	$4 - 10 + 5 = -1$
3	$9 - 15 + 5 = -1$
4	$16 - 20 + 5 = 1$
5	$25 - 25 + 5 = 5$

$\left(x - \frac{5}{2}\right)^2 + 5 - \frac{25}{4}$

Min at $\left(\frac{5}{2}, -\frac{5}{4}\right)$