MEI CORE 1 JUNE 2006

1) $V = \frac{1}{3}\pi r^2 h$

$3V = \pi r^2 h$

$\frac{3V}{\pi h} = r^2$

$r = \pm\sqrt{\frac{3V}{\pi h}}$

2) $x^3 + ax^2 + 7 = 0$

$x = -2$ a root so

$(-2)^3 + a(-2)^2 + 7 = 0$

$-8 + 4a + 7 = 0$

$4a = 8 - 7 = 1$

$a = \frac{1}{4}$

3) $3x + 2y = 6$

Line parallel of form

$3x + 2y = c$

$(2, 10)$ on line

$\therefore 3 \times 2 + 2 \times 10 = c$

$26 = c$

Line is $3x + 2y = 26$

4)

i) $P: x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = 1$ or $x = -2$

$Q: x = 1$

$P \Leftarrow Q$

4ii) $P: y^3 > 1$

$Q: y > 1$

$P \Leftrightarrow Q$

5) $y = 3x + 1$ ①

$x + 3y = 6$ ②

Subst for y in ②

$x + 3(3x+1) = 6$

$x + 9x + 3 = 6$

$10x = 6 - 3$

$x = \frac{3}{10}$

Subst for x in ①

$y = 3 \times \frac{3}{10} + 1$

$y = 1\frac{9}{10}$

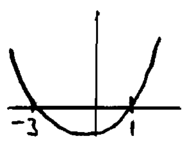
Intersect at $(0.3, 1.9)$

6) $x^2 + 2x < 3$

$x^2 + 2x - 3 < 0$

$(x+3)(x-1) < 0$

$y = x^2 + 2x - 3$



$-3 < x < 1$

7) i) $6\sqrt{2} \times 5\sqrt{3} - \sqrt{24}$

$= 30\sqrt{6} - \sqrt{4 \times 6}$

$= 30\sqrt{6} - 2\sqrt{6} = 28\sqrt{6}$

7ii) $(2 - 3\sqrt{5})^2$

$= (2 - 3\sqrt{5})(2 - 3\sqrt{5})$

$= 4 - 6\sqrt{5} - 6\sqrt{5} + 9 \times 5$

$= 49 - 12\sqrt{5}$

8 $^6C_3 = \dfrac{\cancel{6} \times 5 \times 4}{\cancel{3} \times \cancel{2} \times 1} = 20$

$(1 - 2x)^6$

Term in x^3

$= {}^6C_3 (1)^3 (-2x)^3$

$= 20 \times (-8x^3)$

$= -160x^3$

Coefficient $= -160$

9) i) $\dfrac{16^{\frac{1}{2}}}{81^{3/4}} = \dfrac{\sqrt{16}}{(\sqrt[4]{81})^3} = \dfrac{4}{27}$

ii) $\dfrac{12(a^3b^2c)^4}{4a^2c^6} = \dfrac{12a^{12}b^8c^4}{4a^2c^6}$

$= \dfrac{3a^{10}b^8}{c^2}$

10) $x^2 + y^2 = 25$ ①

$y = 3x$ ②

Subst for y in ①

$x^2 + (3x)^2 = 25$

$10x^2 = 25$

$x^2 = \dfrac{25}{10} = \dfrac{5}{2}$

$x = \pm\sqrt{\dfrac{5}{2}}$

Subst for x in ②

$y = \pm 3\sqrt{5/2}$

Points of intersection

$\left(\sqrt{5/2}, 3\sqrt{5/2}\right)$ and $\left(-\sqrt{5/2}, -3\sqrt{5/2}\right)$

11) A(9, 8)
B(5, 0)
C(3, 1)

i) Gradient AB $= \dfrac{8-0}{9-5} = \dfrac{8}{4} = 2$

Gradient BC $= \dfrac{0-1}{5-3} = \dfrac{-1}{2} = -\frac{1}{2}$

AB and BC are $\perp$ since $2 \times -\frac{1}{2} = -1$

11 ii) Midpoint of AC $= \left(\dfrac{9+3}{2}, \dfrac{8+1}{2}\right)$

Midpoint M $= (6, 4.5)$

11 ii) cont)
Distance AM

$= \sqrt{(9-6)^2 + (8-4.5)^2}$

$= \sqrt{3^2 + 3.5^2}$

$= \sqrt{9 + 12.25} = \sqrt{21.25}$

Circle is given by

$(x-6)^2 + (y-4.5)^2 = 21.25$

B (5, 0)

$(5-6)^2 + (0-4.5)^2$

$= 1 + 20.25 = 21.25$

$\therefore$ B lies on circle

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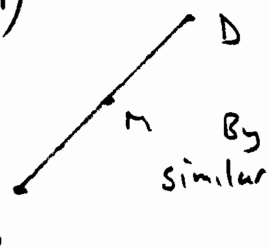
11 iii)

B (5, 0)
M (6, 4.5)

so D (7, 9)



By similar Δs

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12)

$f(x) = x^3 + 9x^2 + 20x + 12$

i) $f(-2) = (-2)^3 + 9(-2)^2 + 20(-2) + 12$

$= -8 + 36 - 40 + 12 = 0$

$\therefore x = -2$ a root of $f(x) = 0$

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ii)

$$\begin{array}{r|l} & x^2 + 3x + 2 \\ x+6 & x^3 + 9x^2 + 20x + 12 \\ & \underline{x^3 + 6x^2} \\ & 3x^2 + 20x \\ & \underline{3x^2 + 18x} \\ & 2x + 12 \\ & \underline{2x + 12} \end{array}$$

Answer $= x^2 + 3x + 2$

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12 iii)

$f(x) = (x+6)(x^2+3x+2)$

$f(x) = (x+6)(x+2)(x+1)$

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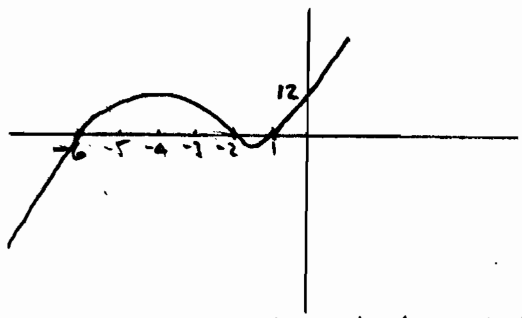
12 iv)



Cuts x-axis at $(-6,0)$, $(-2,0)$, $(-1,0)$

Cuts y-axis at $(0,12)$

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12 v)

$f(x) = 12$

$x^3 + 9x^2 + 20x + 12 = 12$

$x^3 + 9x^2 + 20x = 0$

$x(x^2 + 9x + 20) = 0$

$x(x+5)(x+4) = 0$

$\Rightarrow x = 0$
or $x = -5$
or $x = -4$

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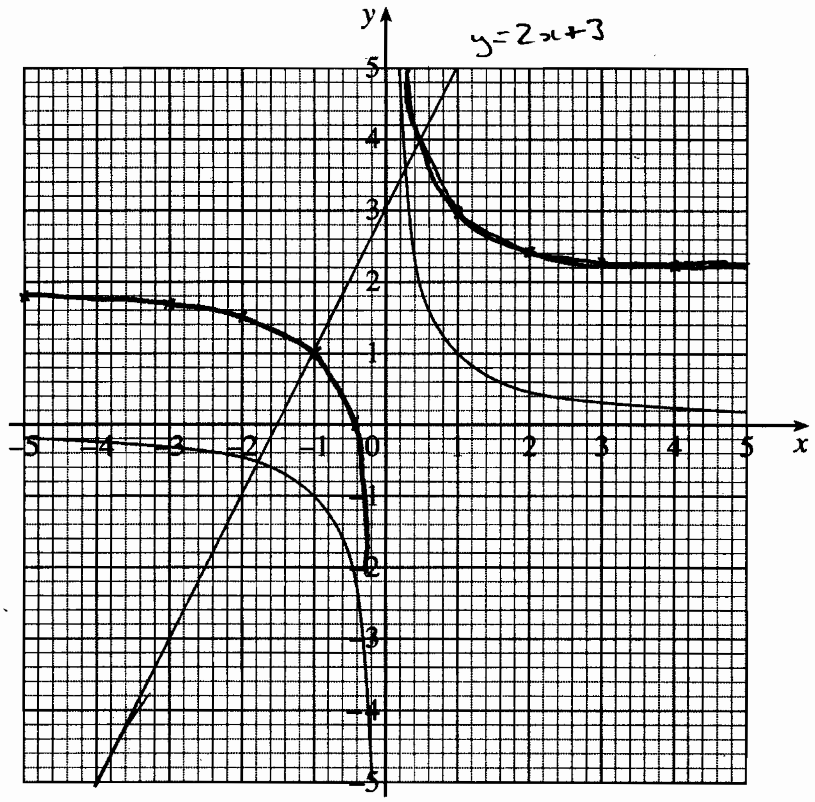
Draw $y = 2x+3$
$x=0,\ y=3$
$x=1,\ y=5$

13 (i) and (iii)



Roots of $\frac{1}{x} = 2x+3$

$x = -1.8$ and $x = 0.3$

(ii) $\frac{1}{x} = 2x+3$

$1 = 2x^2+3x$

$2x^2+3x-1=0$

$x = \frac{-3 \pm \sqrt{3^2+8}}{4}$

$x = \frac{-3 \pm \sqrt{17}}{4}$

(iv) $x = -1$ and $x = 0.5$