MEI CORE 1 MAY 2011

1) $6(x+3) > 2x+5$

$6x + 18 > 2x + 5$

$6x - 2x > +5 - 18$

$4x > -13$

$x > -\frac{13}{4}$

2) $m = \frac{y_2 - y_1}{x_2 - x_1}$

$3 = \frac{k - -5}{5 - 1}$

$3 = \frac{k+5}{4}$

$12 = k+5$

$7 = k$

$k = 7$

3) i) $\left(\frac{9}{16}\right)^{-\frac{1}{2}} = \left(\frac{16}{9}\right)^{\frac{1}{2}} = \frac{4}{3}$

ii) $\frac{(2ac^2)^3 \times 9a^2c}{36a^4c^{12}}$

$= \frac{8a^3c^6 \times 9a^2c}{36a^4c^{12}}$

$= \frac{72a^5c^7}{36a^4c^{12}}$

$= \frac{2a}{c^5}$ or $2ac^{-5}$

4) i) $P(5,4)$ on $y = f(x)$

$y = f(x-5)$

$P(5,4) \rightarrow (10,4)$

ii) $y = f(x) + 7$

$P(5,4) \rightarrow (5,11)$

5) Find coefficient of x^4 in $(5+2x)^6$

Term in x^4 is given by

${}^6C_4\ 5^2 \times (2x)^4$

$= \frac{6.5}{2.1} \times 25 \times 16x^4$

$= 15 \times 25 \times 16x^4$

$= 15 \times 400x^4$

$= 6000x^4$

Coefficient of x^4 is 6000

6) $(2x+5)(x-1)(x+3)$

$= (2x+5)(x^2+2x-3)$

$= 2x^3 + 5x^2 + 4x^2 + 10x - 6x - 15$

$= 2x^3 + 9x^2 + 4x - 15$

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7) $3x^2 + 5x + 2$

discriminant $= b^2 - 4ac$

$= 5^2 - 4 \times 3 \times 2$

$= 1$

discriminant > 0

$\therefore$ 2 real roots of $3x^2 + 5x + 2 = 0$

8) $y = \dfrac{1 - 2x}{x + 3}$

$y(x+3) = 1 - 2x$

$yx + 3y = 1 - 2x$

$yx + 2x = 1 - 3y$

$x(y+2) = 1 - 3y$

$x = \dfrac{1 - 3y}{y + 2}$

9) Any line parallel to

$x + 2y = 6$

can be written as

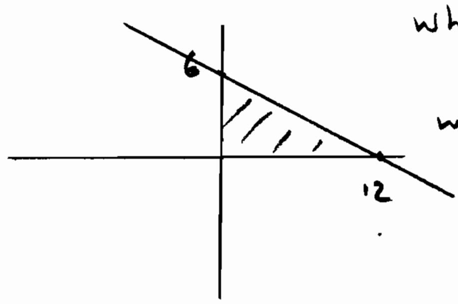
$x + 2y = k$

$(10, 1)$ on this line so

$10 + 2(1) = k$

$\Rightarrow k = 12$

Line is $x + 2y = 12$



when $x = 0$
$y = 6$

when $y = 0$
$x = 12$

Area is area of triangle

$= \frac{1}{2} \times 12 \times 6 = 36$ units^2

10) $n^3 + 3n^2 + 2n$

$= n(n^2 + 3n + 2)$

$= n(n+2)(n+1)$

$= n(n+1)(n+2)$

These are 3 consecutive positive integers if n is a positive integer.

$\therefore$ one is a multiple of 3 and at least one is a multiple of 2.

$\therefore n(n+1)(n+2)$ is divisible by 2 and by 3.

Since 2 and 3 are coprime $n(n+1)(n+2)$ is divisible by $2 \times 3 = 6$

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11) i)

$$y = 4x^2 + 24x + 31 \quad (1)$$

$$x + y = 10 \quad (2)$$

from (2) $y = 10 - x$

Sub in (1)

$$10 - x = 4x^2 + 24x + 31$$

$$0 = 4x^2 + 25x + 21$$

$4 \times 21 = 84$

1×84

2×42

3×28

4×21 ✓

$$0 = 4x^2 + 4x + 21x + 21$$

$$0 = 4x(x+1) + 21(x+1)$$

$$0 = (4x+21)(x+1)$$

$$\Rightarrow \quad x = -\frac{21}{4} \quad \text{or} \quad x = -1$$

when $x = -\frac{21}{4}$

$$-\frac{21}{4} + y = 10$$

$$y = 10 + \frac{21}{4}$$

$$y = \frac{61}{4}$$

$\left(-\frac{21}{4}, \frac{61}{4}\right)$ is a point of intersection

when $x = -1$, $-1 + y = 10$

$y = 11$

$(-1, 11)$ is the other point of intersection

ii)

$$4x^2 + 24x + 31$$

$$= 4\left[x^2 + 6x + \frac{31}{4}\right]$$

$$= 4\left[(x+3)^2 + \frac{31}{4} - 9\right]$$

$$= 4(x+3)^2 + 31 - 36$$

$$= 4(x+3)^2 - 5$$

iii)

A) Line of symmetry

$$x = -3$$

B) Minimum y-value on curve

$$= -5$$

12 i) $y = 2x + 5$

x	-2	0	1
y	$+1$	5	7

Solve $\frac{4}{x^2} = 2x + 5$ graphically

12 i) cont

$x = -2$

$x = -1.3$

$x = 0.8$

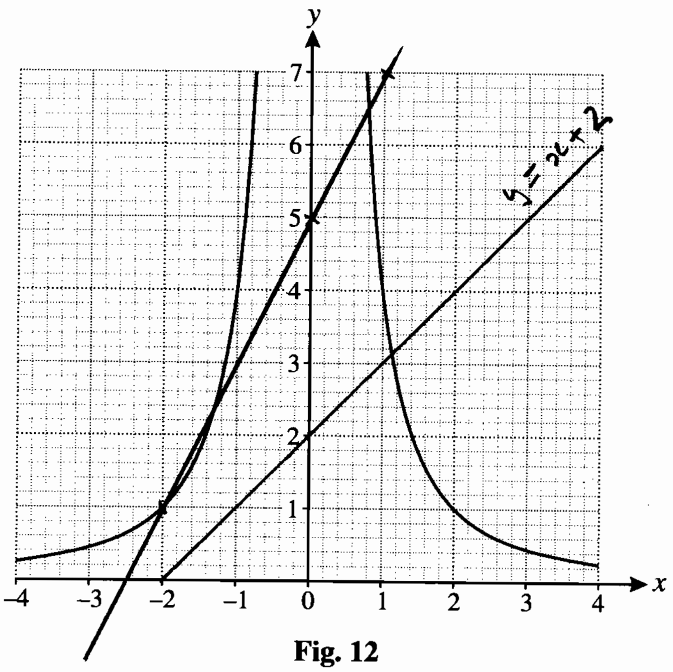

Fig. 12

ii) $\frac{4}{x^2} = 2x + 5$

$4 = 2x^3 + 5x^2$

$0 = 2x^3 + 5x^2 - 4$

$2(-2)^3 + 5(-2)^2 - 4$

$= -16 + 20 - 4 = 0$

$\therefore$ -2 is a root of

$2x^3 + 5x^2 - 4 = 0$

$$\begin{array}{r|l} & 2x^2 + x - 2 \\ \hline x+2 & 2x^3 + 5x^2 \qquad - 4 \\ & \underline{2x^3 + 4x^2} \\ & \qquad + x^2 \\ & \qquad \underline{+ x^2 + 2x} \\ & \qquad\qquad -2x - 4 \\ & \qquad\qquad \underline{-2x - 4} \end{array}$$

$(x+2)(2x^2 + x - 2) = 0$

Other two roots come from

$2x^2 + x - 2 = 0$

$x = \frac{-1 \pm \sqrt{1^2 + 16}}{4}$

$x = \frac{-1 \pm \sqrt{17}}{4}$

$x = \frac{-1 + \sqrt{17}}{4}$ or $x = \frac{-1 - \sqrt{17}}{4}$

iii) If $x^3 + 2x^2 - 4 = 0$

$\div x^2$ $\quad x + 2 - \frac{4}{x^2} = 0$

$x + 2 = \frac{4}{x^2}$

Draw graph of $y = x + 2$

This has only one point of intersection with $y = \frac{4}{x^2}$

$\therefore$ $x^3 + 2x^2 - 4 = 0$ has only 1 real root.

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13) i)

$$(x-4)^2 + (y-2)^2 = 16$$

Circle centre $(4,2)$

radius 4

ii) Crosses x-axis when $y=0$

$$(x-4)^2 + (0-2)^2 = 16$$

$$(x-4)^2 + 4 = 16$$

$$(x-4)^2 = 12$$

$$x-4 = \pm\sqrt{12}$$

$$x-4 = \pm 2\sqrt{3}$$

$$x = 4 \pm \sqrt{3}$$

so $x = 4+\sqrt{3}$ or $x = 4-\sqrt{3}$

iii) $A(4+2\sqrt{2},\ 2+2\sqrt{2})$

Subst in eqn of circle

$$(4+2\sqrt{2}-4)^2 + (2+2\sqrt{2}-2)^2$$

$$= (2\sqrt{2})^2 + (2\sqrt{2})^2$$

$$= 8 + 8 = 16 \checkmark$$

Point lies on the circle

Gradient of line from centre to A is 1

from $\dfrac{2+2\sqrt{2}-2}{4+2\sqrt{2}-4} = \dfrac{2\sqrt{2}}{2\sqrt{2}}$

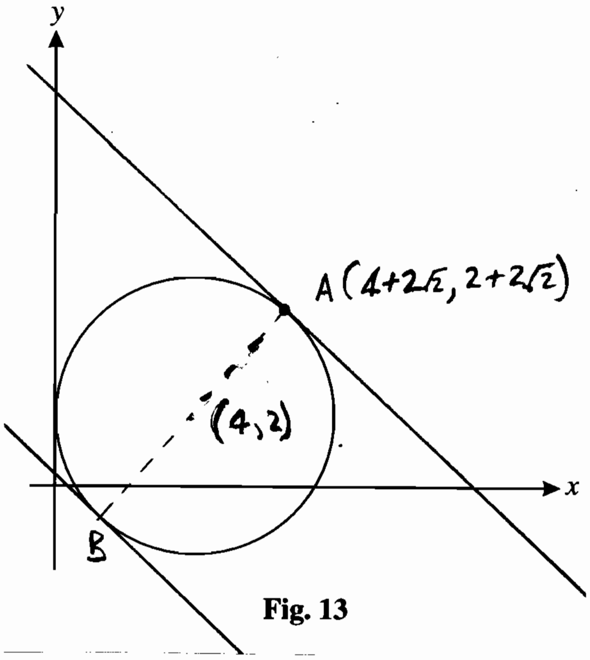

Fig. 13

$\therefore$ gradients of tangents at A and B are -1

$$y - y_1 = m(x - x_1)$$

At A $\quad y - 2 - 2\sqrt{2} = -1(x - 4 - 2\sqrt{2})$

$$y - 2 - 2\sqrt{2} = -x + 4 + 2\sqrt{2}$$

$$y = -x + 6 + 4\sqrt{2}$$

At $B(4-2\sqrt{2},\ 2-2\sqrt{2})$

$$y - y_1 = m(x - x_1)$$

$$y - 2 + 2\sqrt{2} = -1(x - 4 + 2\sqrt{2})$$

$$y - 2 + 2\sqrt{2} = -x + 4 - 2\sqrt{2}$$

$$y = -x + 6 - 4\sqrt{2}$$