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1. $\int_2^5 (2x^3+3)\,dx$

$= \left[\frac{2x^4}{4} + 3x\right]_2^5$

$= \left[\frac{x^4}{2} + 3x\right]_2^5$

$= \left(\frac{5^4}{2} + 3(5)\right) - \left(\frac{2^4}{2} + 3(2)\right)$

$= \left(\frac{625}{2} + 15\right) - (8+6)$

$= 312\frac{1}{2} + 15 - 14$

$= 313\frac{1}{2}$

2. $u_1 = 10$, $u_{r+1} = \frac{5}{u_r^2}$

$u_2 = \frac{5}{10^2} = \frac{5}{100} = \frac{1}{20}$

$u_3 = \frac{5}{\left(\frac{1}{20}\right)^2} = 5 \times 20^2$

$= 2000$

$u_4 = \frac{5}{2000^2} = 0.00000125$

$u_2 = 0.05$, $u_3 = 2000$

$u_4 = 0.00000125$

As $r \to \infty$ the sequence oscillates with alternate terms tending to 0 and alternate terms tending towards infinity

3. $y = \sqrt{1+2x}$

i) when $x = 4$, $y = \sqrt{1+8}$

$y = 3$

when $x = 4.1$ $y = \sqrt{1+8.2}$

$= 3.033150178$

Gradient of chord $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3.033150178 - 3}{4.1 - 4}$

$= \frac{0.033150178}{0.1}$

$= 0.3315$ to 4 d.p.

ii) Use $(4, 3)$

and point where $x = 4.05$

$y = \sqrt{1+8.1}$

$= 3.016620626$

Gradient of new chord

$= \frac{3.016620626 - 3}{4.05 - 4}$

$= 0.3324$ to 4 d.p.

Could use any point where $4 < x < 4.1$

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4. $y = ab^x$ passes through $(1,6)$ and $(2,3.6)$

$6 = ab^1 = ab$ ①

$3.6 = ab^2$ ②

② ÷ ①

$$\frac{3.6}{6} = \frac{ab^2}{ab} = b$$

$\therefore\ 0.6 = b$

Sub for b in ①

$6 = a \times 0.6$

$\frac{6}{0.6} = a$

$10 = a$

Answer $a = 10$, $b = 0.6$

5. $y = 8x^4 + 4$

$\frac{dy}{dx} = 32x^3$

When $x = \frac{1}{2}$, $\frac{dy}{dx} = 32 \times \left(\frac{1}{2}\right)^3$

$= 4$

$\therefore$ gradient of normal $= -\frac{1}{4}$

When $x = \frac{1}{2}$

$y = 8\left(\frac{1}{2}\right)^4 + 4$

$y = \frac{9}{2}$

Using $y - y_1 = m(x - x_1)$

$y - \frac{9}{2} = -\frac{1}{4}\left(x - \frac{1}{2}\right)$

$y - \frac{9}{2} = -\frac{1}{4}x + \frac{1}{8}$

$y = -\frac{1}{4}x + \frac{1}{8} + \frac{9}{2}$

$y = -\frac{1}{4}x + \frac{37}{8}$

Or $8y = -2x + 37$

$2x + 8y - 37 = 0$

6. $\frac{dy}{dx} = 6\sqrt{x} - 2$

$\frac{dy}{dx} = 6x^{\frac{1}{2}} - 2$

$\Rightarrow\ y = \frac{6x^{3/2}}{3/2} - 2x + c$

$y = \frac{2}{3}.6x^{3/2} - 2x + c$

$y = 4x^{3/2} - 2x + c$

Passes through $(9, 4)$

so $4 = 4(9)^{3/2} - 2(9) + c$

$4 = 108 - 18 + c$

$4 = 90 + c$

$-86 = c$

$\therefore\ y = 4x^{3/2} - 2x - 86$

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7.

$$\tan\theta = 2\sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = 2\sin\theta$$

$$\sin\theta = 2\sin\theta\cos\theta$$

$$0 = 2\sin\theta\cos\theta - \sin\theta$$

$$0 = \sin\theta(2\cos\theta - 1)$$

$\Rightarrow \sin\theta = 0$ or $(2\cos\theta - 1) = 0$

when $\sin\theta = 0$

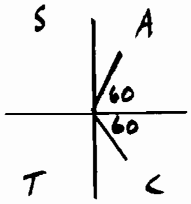
$$\theta = 0^\circ, 180^\circ, 360^\circ$$

when $2\cos\theta - 1 = 0$

$$2\cos\theta = 1$$

$$\cos\theta = \tfrac{1}{2}$$

$$\theta = \cos^{-1}\tfrac{1}{2} = 60^\circ, 300^\circ$$



Solution:

$$\theta = 0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$$

8.

$$p = st^n$$

$$\log_{10} p = \log_{10}(st^n)$$

$$= \log_{10} s + \log_{10} t^n$$

$$= \log_{10} s + n\log_{10} t$$

$$\therefore \log_{10} p - \log_{10} s = n\log_{10} t$$

$$\Rightarrow n = \frac{\log_{10} p - \log_{10} s}{\log_{10} t}$$

9. $\log_a x = \tfrac{1}{2}\log_a 16 + \log_a 75 - 2\log_a 5$

$$\log_a x = \log_a 16^{\frac{1}{2}} + \log_a 75 - \log_a 5^2$$

$$= \log_a 4 + \log_a 75 - \log_a 25$$

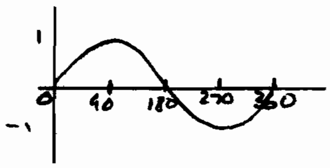
$$= \log_a\left(\frac{4 \times 75}{25}\right)$$

$$\log_a x = \log_a 12$$

$$\Rightarrow x = 12$$

10. $t_n = \sin(\theta + 180n)^\circ$

$$t_1 = \sin(\theta + 180)^\circ$$



$$t_1 = -\sin\theta$$

$$t_2 = \sin(\theta + 360^\circ)$$

$$t_2 = \sin\theta$$

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11.

i) $V = \pi r^2 h$, $A = 2\pi r^2 + 2\pi r h$

If $A = 200$

$$200 = 2\pi r^2 + 2\pi r h$$

$$200 - 2\pi r^2 = 2\pi r h$$

$$\frac{200 - 2\pi r^2}{2\pi r} = h$$

$$\frac{100 - \pi r^2}{\pi r} = h$$

Sub for h in $V = \pi r^2 h$

$$V = \pi r^2 \left(\frac{100 - \pi r^2}{\pi r}\right)$$

$$V = r\left(100 - \pi r^2\right)$$

$$V = 100r - \pi r^3$$

ii)

$$\frac{dV}{dr} = 100 - 3\pi r^2$$

$$\frac{d^2V}{dr^2} = -6\pi r$$

iii) At max value $\frac{dV}{dr} = 0$

$$\Rightarrow \quad 100 - 3\pi r^2 = 0$$

$$3\pi r^2 = 100$$

$$r^2 = \frac{100}{3\pi}$$

$$r = 3.25735$$

Note: $\frac{d^2V}{dr^2} = -6\pi \times 3.25735$

$< 0 \;\therefore$ a max

When $r = 3.25735$

$$V = 100 \times 3.25735 - \pi \times 3.25735^3$$

$$V = 217.16$$

$V = 217$ to 3 s.f.

12. i) AP $a = £500$

$d = -£10$

A) 12^{th} payment $= a + 11d$

$= £500 + 11(-10)$

$= £390$

B)

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_{24} = \frac{24}{2}\left(1000 + 23(-10)\right)$$

$$= 12\left(770\right)$$

$$= £9240$$

12 ii)

A) GP $a = £460$

$r = 0.98$

12^{th} payment $= ar^{11}$

$= £460 \times 0.98^{11}$

$= £368.34$

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12ii) B)

Jim 20^{th} $= a + 19d$

$= 500 + 19(-10)$

$= £310$

Mary 20^{th} $= 460 \times 0.98^{19}$

$= £313.37$

$£310 < £313.37$
Jim Mary

Jim 19^{th} $= a + 18d$

$= 500 + 18(-10)$

$= £320$

Mary 19^{th} $= 460 \times 0.98^{18}$

$= £319.76$

$£320 > £319.76$
Jim Mary

C)

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{24} = \frac{460(1-0.98^{24})}{1-0.98}$$

$$S_{24} = £8837.05$$

D)

$$a = \frac{S_n(1-r)}{(1-r^n)}$$

$$a = \frac{9240(1-0.98)}{(1-0.98^{24})}$$

$$a = £480.97$$

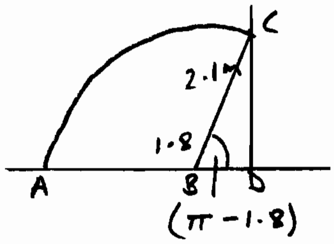
13. i) Arc $= r\theta$

$= 2.1 \times 1.8$

$= 3.78\text{m}$

Area of polythene $= 3.78 \times 5.5$

$= 20.79 \text{ m}^2$



$$\cos(\angle DBC) = \frac{BD}{BC}$$

$\therefore BD = BC \times \cos(\angle DBC)$

$= 2.1 \times \cos(\pi - 1.8)$

$= 0.477\text{m}$ to 3 s.f.

Area of sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 2.1^2 \times 1.8$

$= 3.969 \text{ m}^2$

Area of $\Delta = \frac{1}{2} ab\sin C$

$= \frac{1}{2} \times 2.1 \times 0.477 \sin(\pi - 1.8)$

$= 0.488 \text{ m}^2$

Volume $= (3.969 + 0.488) \times 5.5 \text{ m}^3$

$= 24.5 \text{ m}^3$ to 3 s.f.