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1) $\sum_{r=3}^{6} r(r+2)$

$= 3\times5 + 4\times6 + 5\times7 + 6\times8$

$= 15 + 24 + 35 + 48$

$= 122$

2) $\int \left(x^5 + 10x^{3/2}\right) dx$

$= \dfrac{x^6}{6} + \dfrac{10x^{5/2}}{5/2} + c$

$= \dfrac{x^6}{6} + 4x^{5/2} + c$

3) Let $y = x^2 - 7x$

$\dfrac{dy}{dx} = 2x - 7$

y is a decreasing function when $\dfrac{dy}{dx} < 0$

$\Rightarrow \quad 2x - 7 < 0$

$2x < 7$

$x < \frac{7}{2}$

4) $a > 0$

i) $\log_a 1 = 0$

ii) $\log_a (a^3)^6 = \log_a (a^{18})$

$= 18 \log_a a = 18$

iii) $\log_a \sqrt{a} = \log_a a^{\frac{1}{2}}$

$= \frac{1}{2} \log_a a = \frac{1}{2}$

5) i) $y = 2\sin x$

ii) $y = \sin\left(\frac{x}{2}\right)$

6) $235 \times 5^x = 987$

$5^x = \dfrac{987}{235}$

$\log_{10} 5^x = \log_{10}\left(\dfrac{987}{235}\right)$

$x \log_{10} 5 = \log_{10}\left(\dfrac{987}{235}\right)$

$x = \dfrac{\log_{10}\left(\frac{987}{235}\right)}{\log_{10} 5}$

$x = 0.891668$

$x = 0.892$ to 3 d.p.

7) $y = a + x^b$

$y - a = x^b$

$\log_{10}(y-a) = \log_a x^b$

$\log_{10}(y-a) = b \log_{10} x$

$\log_{10} x = \dfrac{\log_{10}(y-a)}{b}$

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8) $4\cos^2\theta = 1 + \sin\theta$

$4(1 - \sin^2\theta) = 1 + \sin\theta$

$4 - 4\sin^2\theta = 1 + \sin\theta$

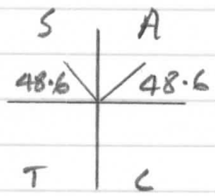
$0 = 1 + \sin\theta - 4 + 4\sin^2\theta$

$0 = 4\sin^2\theta + \sin\theta - 3$

$0 = (4\sin\theta - 3)(\sin\theta + 1)$

$\Rightarrow \sin\theta = \frac{3}{4}$ or $\sin\theta = -1$

$\sin^{-1}\frac{3}{4} = 48.6^\circ$



$\theta = 48.6^\circ, 131.4^\circ$

Also when $\sin\theta = -1$

$\theta = 270^\circ$

9) GP $r > 0$

$a = 32$

$ar = b$

$ar^2 = 12.5$

$\frac{ar^2}{a} = r^2 = \frac{12.5}{32} = \frac{25}{64}$

$\Rightarrow r = \sqrt{\frac{25}{64}} = \frac{5}{8}$

$b = ar = 32 \times \frac{5}{8} = 20$

$b = 20$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{15} = \frac{32\left(\left(\frac{5}{8}\right)^{15} - 1\right)}{\frac{5}{8} - 1}$

$= 85.2593$

$= 85.26$ to 4 s.f.

10) AP 2nd $a + d = 11$ (1)

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{40} = \frac{40}{2}(2a + 39d) = 3030$

$20(2a + 39d) = 3030$

$2(2a + 39d) = 303$

$4a + 78d = 303$ (2)

From (1) $a = 11 - d$

Sub for a in (2)

$4(11 - d) + 78d = 303$

$44 - 4d + 78d = 303$

$74d = 303 - 44$

$74d = 259$

$d = \frac{259}{74} = 3.5$

$a = 11 - d = 11 - 3.5 = 7.5$

$a = 7.5, \ d = 3.5$

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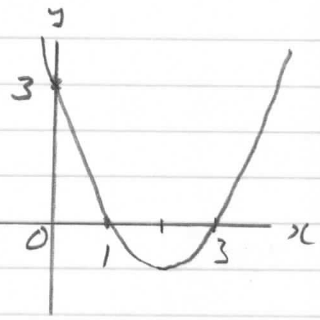
11) i) $y = x^2 - 4x + 3$

when $x = 5$, $y = 5^2 - 4(5) + 3$

$y = 8$

so $A(5,8)$

$y = (x-1)(x-3)$



ii) From above $A(5,8)$

$\frac{dy}{dx} = 2x - 4$

When $x = 5$, $\frac{dy}{dx} = 2(5) - 4 = 6$

tgt $y - y_1 = m(x - x_1)$

$y - 8 = 6(x - 5)$

$y - 8 = 6x - 30$

$y = 6x - 22$

iii) gradient of normal $= -\frac{1}{6}$

$y - y_1 = m(x - x_1)$

$y - 8 = -\frac{1}{6}(x - 5)$

$6y - 48 = -(x - 5)$

$6y - 48 = -x + 5$

$x + 6y = 5 + 48$

$x + 6y = 53$

Solve $\left.\begin{array}{l} x + 6y = 53 \\ y = x^2 - 4x + 3 \end{array}\right\}$ ① ②

Sub for y in ①

$x + 6(x^2 - 4x + 3) = 53$

$x + 6x^2 - 24x + 18 = 53$

$6x^2 - 23x - 35 = 0$

$$\begin{array}{r|l} & 6x + 7 \\ \hline x - 5 & 6x^2 - 23x - 35 \\ & 6x^2 - 30x \\ \hline & \quad 7x - 35 \\ & \quad 7x - 35 \\ \hline \end{array}$$

since root at $x = 5$

$(x - 5)(6x + 7) = 0$

$\Rightarrow x = 5$ or $x = -\frac{7}{6}$

x-coord $= -\frac{7}{6}$ when normal meets curve again

12) $y = 9x^2 - x^4$

i) Meets x-axis when $y = 0$

$0 = 9x^2 - x^4$

$0 = x^2(9 - x^2)$

$0 = x^2(3 + x)(3 - x)$

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12 i) cont)

$\Rightarrow x = 0, \; x = \pm 3$

So meets at origin and at $\pm a$ where $a = 3$

ii)

$y = 9x^2 - x^4$

$\frac{dy}{dx} = 18x - 4x^3$

$\frac{d^2y}{dx^2} = 18 - 12x^2$

st pt when $\frac{dy}{dx} = 0$

$\Rightarrow 18x - 4x^3 = 0$

$2x(9 - 2x^2) = 0$

$2x(3 + \sqrt{2}x)(3 - \sqrt{2}x) = 0$

$\Rightarrow x = 0, \; x = \pm \frac{3}{\sqrt{2}}$

when $x = 0$

$\frac{d^2y}{dx^2} = 18 - 0 = 18 > 0$

$\therefore$ minimum at $x = 0$

when $x = \pm \frac{3}{\sqrt{2}}$

$\frac{d^2y}{dx^2} = 18 - 12\left(\frac{9}{2}\right)$

$= -36 < 0$

$\therefore$ maximums when

$x = \frac{3}{\sqrt{2}}$ and $x = -\frac{3}{\sqrt{2}}$

iii)

Area $= \int_0^3 y \, dx$

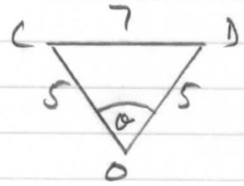
$= \int_0^3 (9x^2 - x^4) \, dx$

$= \left[3x^3 - \frac{x^5}{5}\right]_0^3$

$= \left(3(3)^3 - \frac{3^5}{5}\right) - (0 - 0)$

$= 81 - \frac{243}{5} = 32.4$ units^2

13)
i)



Cosine Rule

$\cos\theta = \frac{5^2 + 5^2 - 7^2}{2 \times 5 \times 5} = \frac{1}{50}$

$\cos^{-1}\left(\frac{1}{50}\right) = 1.55079$ radians

$= 1.55$ to 2 d.p

Area of stage

$= $ area of sector $-$ area of $\triangle$

$= \frac{1}{2}r^2\theta - \frac{1}{2} \times 5^2 \sin\theta$

$= \frac{1}{2} \times 11^2 \times 1.55 - \frac{1}{2} \times 5^2 \sin 1.55$

$= 81.2777 \text{ m}^2$

$= 81.3 \text{ m}^2$ to 3 s.f.

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13 ii) A) Angle of seating arc

$= 2\pi - 1.55$

$= 4.73$ radians

Front row arc $= r\theta$

$= 7.4 \times 4.73$

$= 35.002$ m

Number of front row seats

$= \frac{35.002}{0.8} = 43.75$

so 43 seats in front row

B) Back row arc $= r\theta$

$= 11 \times 4.73$

$= 52.03$ m

Number of back row seats

$= \frac{52.03}{0.8} = 65.04$

so 65 back row seats

$65 - 43 = 22$

so 22 more seats in back row than in front row