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1) $\int 30x^{3/2} = \dfrac{30x^{5/2}}{5/2} + c$

$= \frac{2}{5} \times 30x^{5/2} + c$

$= 12x^{5/2} + c$

2) i) $3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}$ convergent halving each time

ii) $3, 7, 11, 15$ divergent adding 4 each time

iii) $3, 5, -3, -5, 3, 5, -3, -5,$ periodic repeats every 4 terms

3) i) $P(4,-2)$ on $y = f(x)$

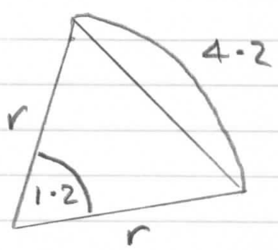
$y = f(5x) \Rightarrow P'\left(\frac{4}{5}, -2\right)$

ii)

$y = \sin x \rightarrow y = \sin(x - 90°)$

Translation by $\begin{pmatrix} 90° \\ 0 \end{pmatrix}$

4)



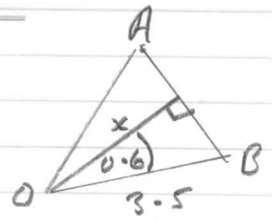
i) Arc length $= r\theta$

$4.2 = 1.2r$

$\dfrac{4.2}{1.2} = r$

$r = 3.5$ cm

ii)



$\cos 0.6 = \dfrac{x}{3.5}$

$3.5 \cos 0.6 = x$

$x = 2.89$ cm

$\perp$ distance of chord AB from O

$= 2.89$ cm

5) $y = 4\sqrt{x}$ $A(9, 12)$

$B(9.5, ?)$

When $x = 9.5$, $y = 4\sqrt{9.5} = 2\sqrt{38}$

Gradient of AB $\dfrac{2\sqrt{38} - 12}{9.5 - 9}$

$= 0.658$ to 3 s.f.

Any point closer to A than B is will do

Say $x = 9.2$ for C

6) $\dfrac{d}{dx}\left(2x^3 + 9x^2 - 24x\right)$

$= 6x^2 + 18x - 24$

$f(x)$ increasing when $f'(x) > 0$

$\Rightarrow 6x^2 + 18x - 24 > 0$

$\Rightarrow x^2 + 3x - 4 > 0$

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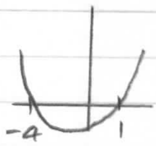
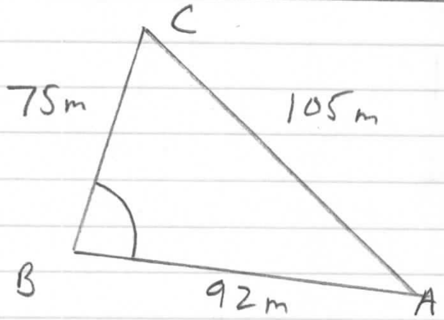
6 cont)

$(x+4)(x-1) > 0$



Either $x < -4$ or $x > 1$

7)



Cosine rule

$$\cos B = \frac{75^2 + 92^2 - 105^2}{2 \times 75 \times 92}$$

$$\Rightarrow B = 77.172^\circ$$

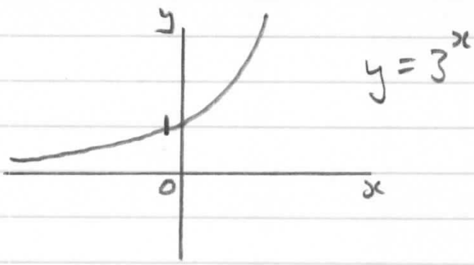
$$\text{Area} = \frac{1}{2} ac \sin B$$

$$= \frac{1}{2} \times 75 \times 92 \sin 77.172^\circ$$

$$= 3363.89 \text{ m}^2$$

$$= 3364 \text{ m}^2 \text{ to 4 s.f.}$$

8) i)



ii) $3^{5x-1} = 500000$

$$\log_{10} 3^{5x-1} = \log_{10} 500000$$

$$(5x-1)\log_{10} 3 = \log_{10} 500000$$

$$5x - 1 = \frac{\log_{10} 500000}{\log_{10} 3}$$

$$5x = \frac{\log_{10} 500000}{\log_{10} 3} + 1$$

$$x = \frac{\frac{\log_{10} 500000}{\log_{10} 3} + 1}{5}$$

$$x = 2.588898$$

$$x = 2.59 \text{ to 3 s.f.}$$

9) i) $\frac{\tan\theta}{\cos\theta} = 1$

$$\tan\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \cos\theta$$

$$\sin\theta = \cos^2\theta$$

$$\underline{\sin\theta = 1 - \sin^2\theta}$$

ii) $\sin\theta = 1 - \sin^2\theta$

$$\sin^2\theta + \sin\theta - 1 = 0$$

$$\sin\theta = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-1)}}{2}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

$\sin\theta = 0.618034$ or $\cancel{-1.618}$ impossible

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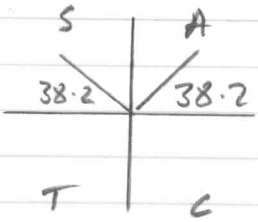
9ii) cont)

$\sin^{-1}(0.618034) = 38.2^\circ$



$\theta = 38.2^\circ,\ 141.8^\circ$

10) $y = x^2 - 4x + 3$

i) when $x = 4$, $y = 4^2 - 4(4) + 3$
$= 3$

so $A(4, 3)$

$\frac{dy}{dx} = 2x - 4$

when $x = 4$, $\frac{dy}{dx} = 2(4) - 4$
$= 8 - 4 = 4$

$\therefore$ normal has gradient $-\frac{1}{4}$

Normal $y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{4}(x - 4)$

$4y - 12 = -1(x - 4)$

$4y - 12 = -x + 4$

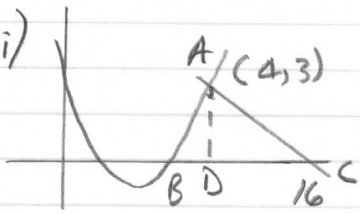
$x + 4y - 12 - 4 = 0$

$x + 4y - 16 = 0$

When $y = 0$, $x - 16 = 0$
$x = 16$

So normal intersects x-axis at $(16, 0)$

ii)



Find B, $y = 0$

$0 = x^2 - 4x + 3$
$0 = (x - 3)(x - 1)$

$\Rightarrow x = 3$ or $x = 1$

So $B(3, 0)$

Area ABC

$= \int_3^4 y\,dx$ + Area of $\triangle ADC$

$= \int_3^4 (x^2 - 4x + 3)\,dx + \frac{1}{2} \times 12 \times 3$

$= \left[\frac{x^3}{3} - 2x^2 + 3x\right]_3^4 + 18$

$= \left(\frac{4^3}{3} - 2(4)^2 + 3(4)\right) - \left(\frac{3^3}{3} - 2(3)^2 + 3(3)\right) + 18$

$= \frac{4}{3} - 0 + 18$

$= 19\frac{1}{3}$ units^2

or $\frac{58}{3}$ units^2

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11) i) AP $a = A$ $d = D$

A) $A,\ A+D,\ A+2D,\ A+3D$

first 2 sum $2A + D = 25$ ①

first 4 sum $4A + 6D = 250$ ②

①×2 $4D + 2D = 50$ ③

②−③ $4D = 200$

$\Rightarrow D = 50$

Sub in ① $2A + 50 = 25$
$2A = 25 - 50$
$2A = -25$

$A = -\frac{25}{2}$

so $A = -12.5,\ D = 50$

i) B) $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

Sum of 21st to 50th given by

$S_{50} - S_{20}$

$S_{50} = \frac{50}{2}\left(-25 + 49 \times 50\right)$

$= 60625$

$S_{20} = \frac{20}{2}\left(-25 + 19 \times 50\right)$

$= 9250$

Required sum $= 60625 - 9250$

$= 51375$

11) ii) GP
$a,\ ar,\ ar^2\ ar^3$

$a + ar = 25$ ①

$a + ar + ar^2 + ar^3 = 250$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$$\frac{S_4}{S_2} = \frac{\frac{a(r^4-1)}{r-1}}{\frac{a(r^2-1)}{r-1}} = \frac{250}{25}$$

$$\Rightarrow \frac{a(r^4-1)}{(r-1)} \times \frac{(r-1)}{a(r^2-1)} = 10$$

$$\Rightarrow \frac{r^4-1}{r^2-1} = 10$$

$r^4 - 1 = 10(r^2 - 1)$

$r^4 - 1 = 10r^2 - 10$

$r^4 - 10r^2 - 1 + 10 = 0$

$r^4 - 10r^2 + 9 = 0$

$(r^2 - 9)(r^2 - 1) = 0$

Either $r^2 = 9 \Rightarrow r = \pm 3$

or $r^2 = 1 \Rightarrow r = \pm 1$

Given $r \neq \pm 1$

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11ii) cont) when $r = 3$ $4a = 25$

$a = 6.25$

when $r = -3$ $-2a = 25$

$a = -12.5$

Possible values

$a = 6.25$ and $r = 3$

or $a = -12.5$ and $r = -3$

12i) Next sheet

12ii) Next sheet

12iii) Intercept 1.68 $\Rightarrow \log_{10} a = 1.68$

$a = 10^{1.68} = 47.9$

$k = \text{gradient} = \dfrac{2.26 - 1.68}{50 - 0}$

$= 0.0116$

$p = a \times 10^{kt}$

$p = 47.9 \times 10^{0.0116t}$

Also $\log_{10} p = \log_{10} 1.68 + 0.0116t$

12iv) 1960

$p = 47.9 \times 10^{0}$

$p = 47.9$ million

so population 47.9 million

12v) $p = 47.9 \times 10^{0.0116t}$

$200 = 47.9 \times 10^{0.0116t}$

$\dfrac{200}{47.9} = 10^{0.0116t}$

$\log_{10}\left(\dfrac{200}{47.9}\right) = \log_{10} 10^{0.0116t}$

$\log_{10}\left(\dfrac{200}{47.9}\right) = 0.0116t$

$t = \dfrac{\log_{10}\left(\frac{200}{47.9}\right)}{0.0116}$

$t = 53.5$ years

so $1960 + 53.5 = 2013.5$

During 2014

12 (i)

$p = a \times 10^{kt}$

$\log_{10} p = \log_{10}(a + 10^{kt})$

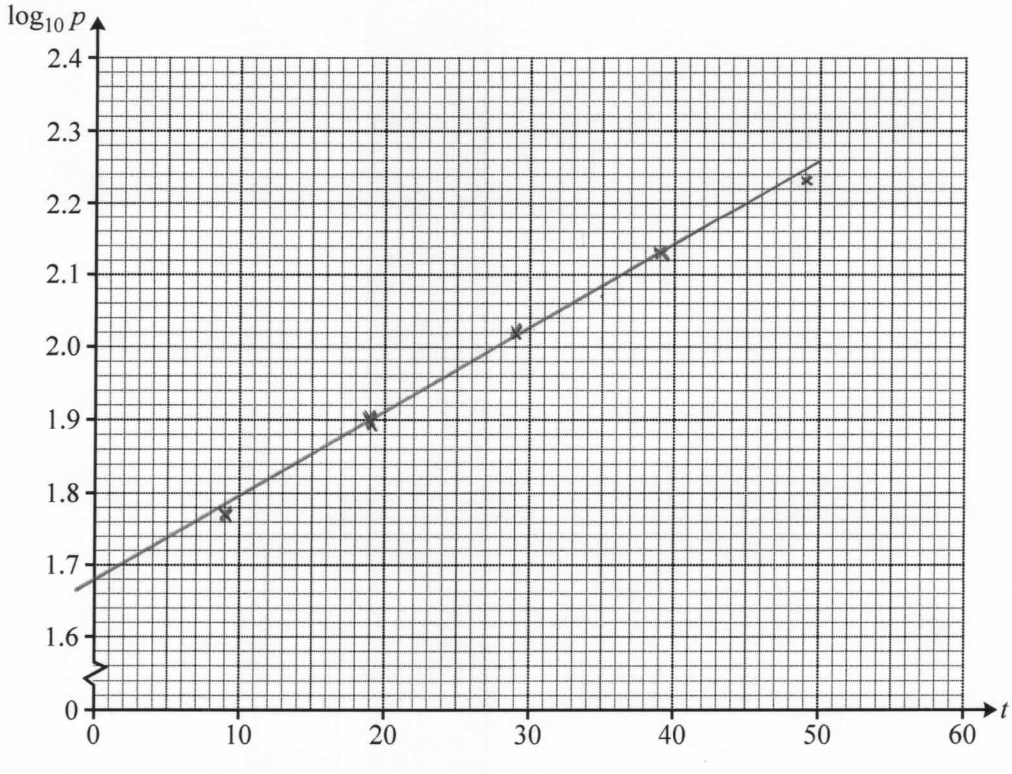
$\log_{10} p = \log_{10} a + \log_{10} 10^{kt}$

$\log_{10} p = \log_{10} a + kt \log_{10} 10$

$\log_{10} p = \log_{10} a + kt$

12 (ii)

Year	1969	1979	1989	1999	2009
Number of years after 1960 (t)	9	19	29	39	49
Population in millions (p)	58.81	80.35	105.27	134.79	169.71
$\log_{10} p$	1.77	1.90	2.02	2.13	2.23



A spare copy of this graph paper can be found on page 12.