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1) $\int 7x^{5/2}\,dx$

$= \dfrac{7x^{7/2}}{7/2} + c$

$= 2x^{7/2} + c$

2) i) $\displaystyle\sum_{r=1}^{5} \frac{21}{r+2}$

$= \dfrac{21}{3} + \dfrac{21}{4} + \dfrac{21}{5} + \dfrac{21}{6} + \dfrac{21}{7}$

$= \dfrac{459}{20}$ or 22.95

ii) $u_1 = a \qquad u_{n+1} = u_n + 5$

This is an AP with $a = a$, $d = 5$

10^{th} term $= a + 9d = a + 9(5)$

$= a + 45$

$S_n = \dfrac{n}{2}\left(2a + (n-1)d\right)$

$S_{10} = \dfrac{10}{2}\left(2a + 9(5)\right)$

$= 5(2a + 45)$

$= 10a + 225$

3) $P(2, 3.6) \qquad Q(2.2, 2.4)$

Gradient $PQ = \dfrac{2.4 - 3.6}{2.2 - 2}$

$= \dfrac{-1.2}{0.2}$

$= -6$

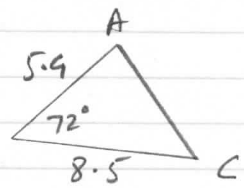
Estimate of gradient at $x = 2$ is -6

4) i) $R(6, -3)$ on $y = f(x)$

$y = \frac{1}{2}f(x) \Rightarrow R'(6, -\frac{3}{2})$

ii) $y = f(3x) \Rightarrow R'(2, -3)$

5)

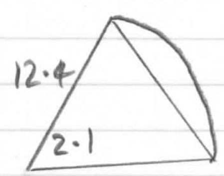


Cosine Rule

$AC^2 = 5.9^2 + 8.5^2 - 2 \times 5.9 \times 8.5 \cos 72°$

$\Rightarrow AC = 8.7$ cm

6) Area of segment
= Area of sector − Area of triangle



$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2} \times 12.4^2(2.1 - \sin 2.1)$

$= 95.08\ \text{cm}^2$

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7) GP

2nd $\quad ar = 24 \quad (1)$

$$S_\infty = \frac{a}{1-r} = 150 \quad (2)$$

From (1) $\quad ar = 24 \Rightarrow a = \frac{24}{r}$

From (2) $\quad a = 150(1-r)$

Sub for a in above

$$\frac{24}{r} = 150(1-r)$$

$$24 = 150r(1-r)$$

$$4 = 25r(1-r)$$

$$4 = 25r - 25r^2$$

$$25r^2 - 25r + 4 = 0$$

25x4
100
-20 -5

$$25r^2 - 20r - 5r + 4 = 0$$

$$5r(5r-4) - 1(5r-4) = 0$$

$$(5r-1)(5r-4) = 0$$

$\Rightarrow \quad r = \frac{1}{5}$ or $r = \frac{4}{5}$

When $r = \frac{1}{5} \quad a = \frac{24}{\frac{1}{5}} = 120$

When $r = \frac{4}{5} \quad a = \frac{24}{\frac{4}{5}} = 30$

Possible values $\quad a = 120, \; r = \frac{1}{5}$

$a = 30, \; r = \frac{4}{5}$

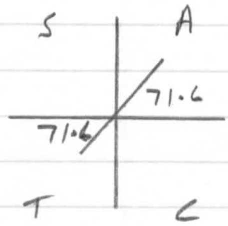
8) $$\frac{\sqrt{1-\cos^2\theta}}{\tan\theta}$$

$$= \frac{\sqrt{\sin^2\theta}}{\frac{\sin\theta}{\cos\theta}}$$

$$= \sin\theta \times \frac{\cos\theta}{\sin\theta} = \cos\theta$$

9) $\tan 2\theta = 3 \quad$ for $0 < \theta < 360°$

$\tan^{-1} 3 = 71.6°$



$2\theta = 71.6°, 251.6°, 431.6°, 611.6°$

$\theta = 35.8°, 125.8°, 215.8°, 305.8°$

10) $$3^{x+1} = 5^{2x}$$

$$\log_{10} 3^{x+1} = \log_{10} 5^{2x}$$

$$(x+1)\log_{10} 3 = 2x \log_{10} 5$$

$$x\log_{10} 3 + \log_{10} 3 = 2x\log_{10} 5$$

$$\log_{10} 3 = 2x\log_{10} 5 - x\log_{10} 3$$

$$\log_{10} 3 = x(2\log_{10} 5 - \log_{10} 3)$$

$$\frac{\log_{10} 3}{(2\log_{10} 5 - \log_{10} 3)} = x$$

$x = 0.518$ to 3 dp

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11) i)

$y = x - \frac{4}{x^2}$

$y = x - 4x^{-2}$

$\frac{dy}{dx} = 1 + 8x^{-3}$

or $1 + \frac{8}{x^3}$

$\frac{d^2y}{dx^2} = -3 \times 8x^{-4}$

$= \frac{-24}{x^4}$

ii)

st pt when $\frac{dy}{dx} = 0$

$\Rightarrow \quad 1 + \frac{8}{x^3} = 0$

$x^3 + 8 = 0$

$x^3 = -8$

$x = \sqrt[3]{-8}$

$x = -2$

when $x = -2$

$y = -2 - \frac{4}{(-2)^2}$

$= -2 - \frac{4}{4}$

$= -2 - 1$

$= -3$

st pt $(-2, -3)$

when $x = -2$ $\frac{d^2y}{dx^2} = \frac{-24}{(-2)^4}$

$= -\frac{24}{16} < 0$

$\therefore$ st pt is a maximum since $\frac{d^2y}{dx^2}$ is negative.

iii)

when $x = -1$, $y = -1 - \frac{4}{(-1)^2}$

$= -1 - 4$

$= -5$

Point is $(-1, -5)$

when $x = -1$ $\frac{dy}{dx} = 1 + \frac{8}{(-1)^3}$

$= 1 - 8 = -7$

So normal has gradient $+\frac{1}{7}$

$y - y_1 = m(x - x_1)$

$y - -5 = \frac{1}{7}(x - -1)$

$y + 5 = \frac{1}{7}(x + 1)$

$7y + 35 = x + 1$

$0 = x - 7y + 1 - 35$

$x - 7y - 34 = 0$

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12) i) $A = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4) + y_5\right]$

$A = \frac{3}{2}\left[9 + 2(10.7 + 11.7 + 11.9 + 11.0) + 9.1\right]$

$\text{Area} = 163.05 \text{ m}^2$

ii) A) $y = -0.001x^3 - 0.025x^2 + 0.6x + 9$

When $x = 12$

$y = -0.001(12)^3 - 0.025(12)^2 + 0.6(12) + 9$

$= 10.872$ m

Previous model gave height 11.0 m

Difference $= 11 - 10.872$

$= 0.128$ m

ii) B) $A = \int_0^{15} \left(-0.001x^3 - 0.025x^2 + 0.6x + 9\right) dx$

$= \left[\frac{-0.001}{4}x^4 - \frac{0.025}{3}x^3 + \frac{0.6x^2}{2} + 9x\right]_0^{15}$

$= \left[\frac{-0.001 \times 15^4}{4} - \frac{0.025 \times 15^3}{3} + \frac{0.6 \times 15^2}{2} + 9 \times 15\right]$

$- \left[0 + 0 + 0 + 0\right]$

$= 161.7 \text{ m}^2$ to 4 s.f.

13) i) Next sheet

ii) Next sheet

13) iii) intercept -0.25

$\log_{10} a = -0.25$

$a = 10^{-0.25} = 0.562$

b is gradient of graph

$= \frac{1.2 - -0.25}{50 - 0} = \frac{1.45}{50}$

$b = 0.029$

$h = a \times 10^{bt}$

$h = 0.562 \times 10^{0.029t}$

13) iv) 2020 $t = 60$

$h = 0.562 \times 10^{0.029 \times 60}$

$h = 30.9$ m

Reduction between 2010 and 2020

$= 30.9 - 15.9$

$= 15$ m

13) v) exponential increase in annual reduction would mean glacier would disappear and model would predict reduction when glacier no longer existed.

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13(i)

$h = a \times 10^{bt}$

$\log_{10} h = \log_{10}(a \times 10^{bt})$

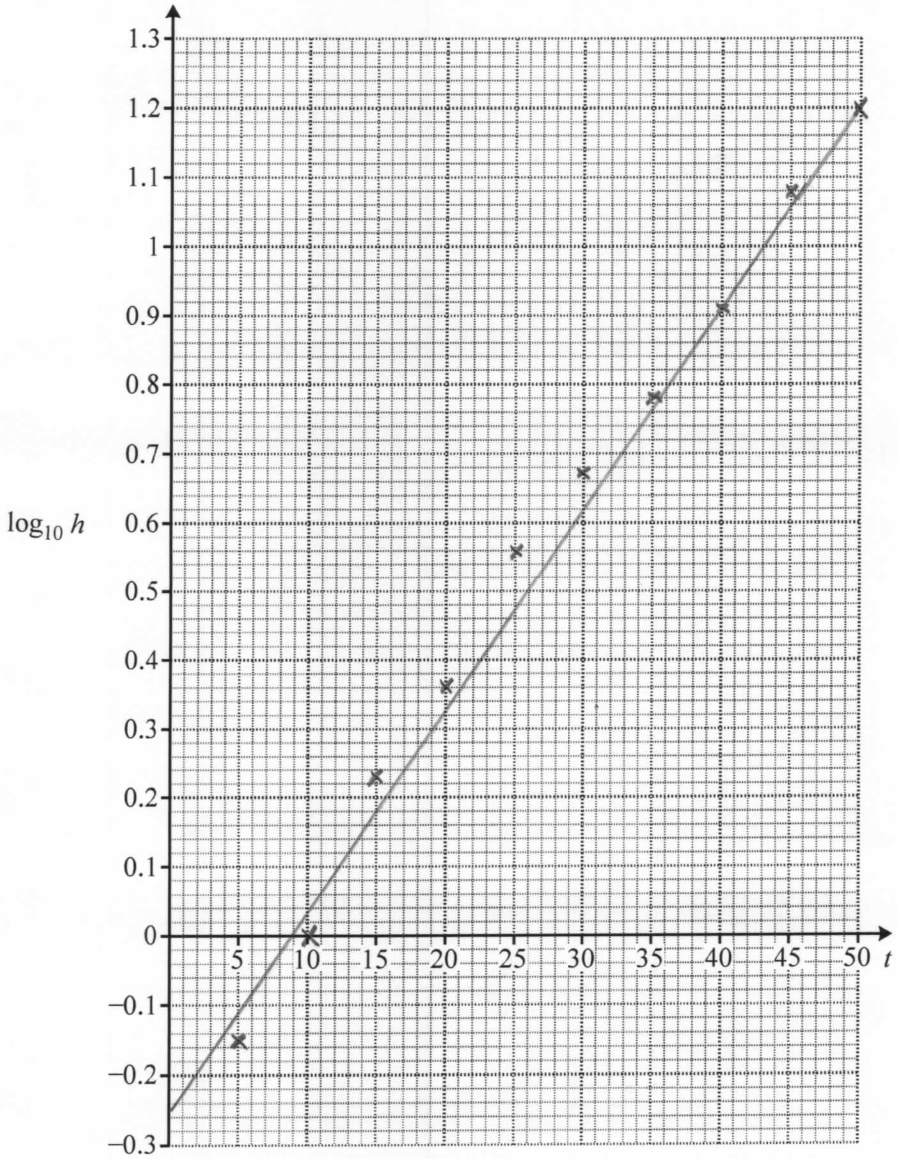
$\log_{10} h = \log_{10} a + \log_{10} 10^{bt}$ so $m = b$

$\log_{10} h = \log_{10} a + bt \log_{10} 10$ and $c = \log_{10} a$

$\log_{10} h = \log_{10} a + bt$

13(ii)

t	5	10	15	20	25	30	35	40	45	50
h	0.7	1.0	1.7	2.3	3.6	4.7	6.0	8.2	12	15.9
$\log_{10} h$	−0.15	0.00	0.23	0.36	0.56	0.67	0.78	0.91	1.08	1.20



A spare copy of this table and graph can be found on page 12.

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