MEI CORE 2 JUNE 2015

1) i) $\frac{d}{dx} 12\sqrt[3]{x}$

$= \frac{d}{dx} 12x^{\frac{1}{3}}$

$= \frac{1}{3} \times 12x^{-2/3}$

$= 4x^{-2/3}$

ii) $\int \frac{6}{x^3} dx = \int 6x^{-3} dx$

$= \frac{6x^{-2}}{-2} + c$

$= -3x^{-2} + c$

or $-\frac{3}{x^2} + c$

2) $u_1 = 2$, $u_{n+1} = \frac{10}{u_n^2}$

$u_2 = \frac{10}{2^2} = 2.5$

$u_3 = \frac{10}{2.5^2} = 1.6$

$u_4 = \frac{10}{1.6^2} = 3.90625$

$\sum_{r=1}^{4} u_r = 2 + 2.5 + 1.6 + 3.90625$

$= 10.00625$

3) 10^{th} $a + 9d = 11.1$ (1)

50^{th} $a + 49d = 7.1$ (2)

(2) − (1) $40d = -4$

$d = -\frac{4}{40} = -0.1$

Sub in (1)

$a + 9(-0.1) = 11.1$

$a - 0.9 = 11.1$

$a = 11.1 + 0.9$

$a = 12$

$a = 12$, $d = -0.1$

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$S_{50} = \frac{50}{2}\left(2(12) + 49(-0.1)\right)$

$= 25(24 - 4.9)$

$= 477.5$

4)

Area $= \frac{1}{2} r^2 \theta$

$27 = \frac{1}{2} \times 1.5 \times r^2$

$\frac{54}{1.5} = r^2$

$36 = r^2 \Rightarrow r = 6$ cm

MEI CORE 2 JUNE 2015

4 cont)

Perimeter $= r + r + r\theta$

$= 6 + 6 + 6 \times 1.5$

$= 21$ cm

---

5) Let $y = x^3 - 6x$

$\frac{dy}{dx} = 3x^2 - 6$

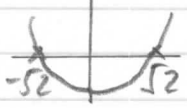
Increasing function when

$\frac{dy}{dx} > 0$

$\Rightarrow 3x^2 - 6 > 0$

$\Rightarrow x^2 - 2 > 0$

$(x + \sqrt{2})(x - \sqrt{2}) > 0$

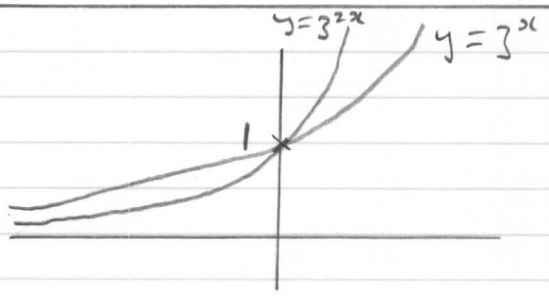


Either $x > \sqrt{2}$

or $x < -\sqrt{2}$

---

6) i)



ii) $3^{2x} = 729$

$3^x \times 3^x = 729$

$\Rightarrow 3^x = \sqrt{729} = 27$

$\Rightarrow x = 3$ since $3^3 = 27$

Answers $x = 3$, $3^x = 27$

---

7) $\sin^2 x = 3\cos x - 2$

$1 - \cos^2 x = 3\cos x - 2$

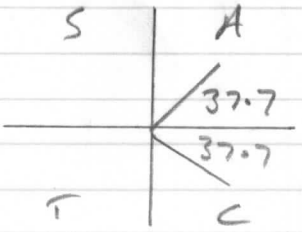
$0 = \cos^2 x + 3\cos x - 3$

$\cos x = \frac{-3 \pm \sqrt{3^2 + 4 \times 1 \times 3}}{2}$

$\cos x = \frac{-3 \pm \sqrt{21}}{2}$

$\cos x = 0.7913 \Rightarrow x = 37.7°$

$\cos x = -3.79$ impossible



$x = 37.7°, 322.3°$

Convert to radians

$x = 37.7 \times \frac{180}{\pi} = 0.658^c$

$x = 322.3 \times \frac{180}{\pi} = 5.625^c$

---

8) $\log_{10} y = m \log_{10} x + c$

Find gradient m

MEI CORE 2 JUNE 2015

8 cont)

$$m = \frac{8-2}{2-0} = \frac{6}{2} = 3$$

Find intercept c

$$c = 2$$

$$\Rightarrow \log_{10} y = 3\log_{10} x + 2$$

$$\log_{10} y = \log_{10} x^3 + 2$$

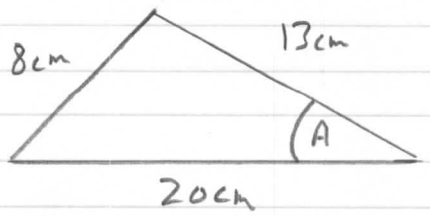
$$\log_{10} y - \log_{10} x^3 = 2$$

$$\log_{10}\left(\frac{y}{x^3}\right) = 2$$

$$\frac{y}{x^3} = 10^2$$

$$y = 100x^3$$

9) i)



cosine rule

$$\cos A = \frac{20^2 + 13^2 - 8^2}{2 \times 20 \times 13}$$

$$\Rightarrow A = 13.8^\circ$$

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$\text{or } \frac{1}{2}bc\sin A$$

$$= \frac{1}{2} \times 20 \times 13 \sin 13.8^\circ = 31.0 \text{ cm}^2$$

ii) $\text{Area} = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4) + y_5\right]$

$y_0 = 0 - 0 = 0$
$y_1 = 1.45 - -0.85 = 2.3$
$y_2 = 1.56 - -0.76 = 2.32$
$y_3 = 1.27 - -0.55 = 1.82$
$y_4 = 1.04 - -0.30 = 1.34$
$y_5 = 0 - 0 = 0$

$$\text{Area} = \frac{4}{2}\left[0 + 2(2.3 + 2.32 + 1.82 + 1.34) + 0\right]$$

$$= 31.12 \text{ cm}^2$$

10) $\frac{dy}{dx} = 4x + 3$ through $(2, 9)$

i) When $x = 2$, $\frac{dy}{dx} = 4(2) + 3 = 11$

tgt $y - y_1 = m(x - x_1)$

$$y - 9 = 11(x - 2)$$

$$y - 9 = 11x - 22$$

$$y = 11x - 22 + 9$$

$$y = 11x - 13$$

ii)

$$y = \int \frac{dy}{dx}\, dx$$

$$y = \int (4x + 3)\, dx$$

$$y = \frac{4x^2}{2} + 3x + c$$

$$y = 2x^2 + 3x + c$$

MEI CORE 2 JUNE 2015

10ii cont) Sub (2, 9)

$9 = 2(2)^2 + 3(2) + c$

$9 = 8 + 6 + c$

$9 - 8 - 6 = c$

$-5 = c$

$\Rightarrow y = 2x^2 + 3x - 5$

Crosses x-axis when $y = 0$

$0 = 2x^2 + 3x - 5$

$0 = (2x + 5)(x - 1)$

$\Rightarrow x = -\frac{5}{2}$ or $x = 1$

Points $\left(-\frac{5}{2}, 0\right)$ and $(1, 0)$

Min point when $\frac{dy}{dx} = 0$

$\Rightarrow 4x + 3 = 0$

$4x = -3$

$x = -\frac{3}{4}$

Then $y = 2\left(-\frac{3}{4}\right)^2 + 3\left(-\frac{3}{4}\right) - 5$

$= \frac{9}{8} - \frac{9}{4} - 5$

$= -\frac{49}{8}$

Min point $\left(-\frac{3}{4}, -\frac{49}{8}\right)$

11) i)

Generation	Descendants	
1	3	$= 3^1$
2	9	$= 3^2$
3	27	$= 3^3$
8	6561	$= 3^8$

Answer 6561

ii) GP $a = 3$ $r = 3$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{15} = \frac{3(3^{15} - 1)}{3 - 1}$

$= 21,523,359$

iii)

$1000000 < \frac{3(3^n - 1)}{3 - 1}$

$2000000 < 3(3^n - 1)$

$2000000 < 3^{n+1} - 3$

$2000003 < 3^{n+1}$

$\log_{10} 2000003 < \log_{10} 3^{n+1}$

$\log_{10} 2000003 < (n+1)\log_{10} 3$

$\frac{\log_{10} 2000003}{\log_{10} 3} < n + 1$

$\Rightarrow n > \frac{\log_{10} 2000003}{\log_{10} 3} - 1$

$n > 12.2$ so least $n = 13$

MEI CORE 2 JUNE 2015

ii) iii)

GP $a = 2$, $r = 2$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{15} = \frac{2(2^{15} - 1)}{2 - 1} = 65534$$

Difference

$21,523,359 - 65,534$

$= 21,457,825$ fewer