MEI CORE 2 JUNE 2016

1) $y = 6\sqrt{x}$

i) $y = 6x^{\frac{1}{2}}$

$\frac{dy}{dx} = 6 \times \frac{1}{2} x^{-\frac{1}{2}}$

$\frac{dy}{dx} = 3x^{-\frac{1}{2}}$ or $\frac{3}{\sqrt{x}}$

ii) $\int \frac{12}{x^2}\,dx = \int 12x^{-2}\,dx$

$= \frac{12x^{-1}}{-1} + c$

$= -\frac{12}{x} + c$

2

i) $u_1 = 5$

$u_2 = 2 \times 5 = 10$

$u_3 = 1 \times 5 = 5$

$u_4 = 10$

$\sum_{r=2}^{4} u_r = 10 + 5 + 10 = 25$

ii) $u_1 = 6$

$u_2 = 0 \times 5 = 0$

$u_3 = 0$

$u_4 = 0$

$\sum_{r=2}^{4} u_r = 0 + 0 + 0 = 0$

3)

	AP	GP	
1st	a	a	1.5
4th	$a+3d$	ar^3	12

GP $\frac{ar^3}{a} = r^3 = \frac{12}{1.5} = 8$

$\Rightarrow r = \sqrt[3]{8} = 2$

AP $a + 3d - a = 3d = 12 - 1.5$

$3d = 10.5$

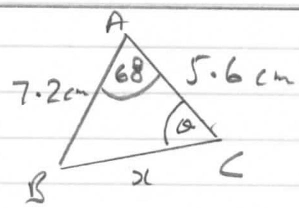
$d = 3.5$

10^{th} $a + 9d = 1.5 + 9(3.5)$ AP
$= 33$

10^{th} $ar^9 = 1.5 \times 2^9$ GP
$= 768$

Difference $= 768 - 33 = 735$

4)



Cosine Rule
$x^2 = 7.2^2 + 5.6^2 - 2 \times 5.6 \times 7.2 \cos 68°$

$\Rightarrow x = 7.2795$

Sine Rule $\frac{7.2}{\sin Q} = \frac{7.2795}{\sin 68°}$

$\frac{\sin Q}{7.2} = \frac{\sin 68°}{7.2795}$

$\sin Q = \frac{\sin 68}{7.2795} \times 7.2 = 0.9171$

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4 cont

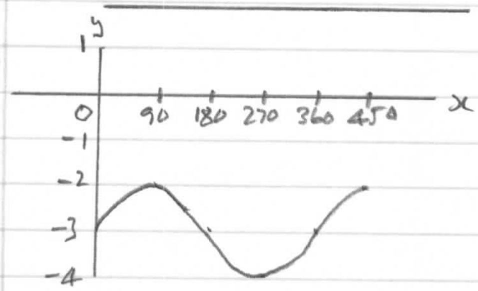
$\theta = \sin^{-1}(0.9171)$

$\theta = 66.5^\circ$

$\angle ACB = 66.5^\circ$

5) i) $y = \sin 2x$

ii)



6) i) Area B $= \frac{1}{2}r^2\theta - \frac{1}{2}a^2\sin\theta$

ii) Area B $= \frac{1}{2} \times 12^2 \times 0.8 - \frac{1}{2}a^2 \sin 0.8$

$= 57.6 - 0.358678a^2$

Area A $= 0.358678a^2$

If Area A = Area B

$57.6 = 2 \times 0.358678a^2$

$\frac{57.6}{0.717356} = a^2$

$80.29486 = a^2$

$a = 8.96$ cm to 3 s.f.

7) i) $\tan x \sqrt{1-\sin^2 x}$

$= \frac{\sin x}{\cos x}\sqrt{\cos^2 x}$

$= \frac{\sin x}{\cos x} \times \cos x = \sin x$

ii) $4\sin^2 y = \sin y$

$4\sin^2 y - \sin y = 0$

$\sin y(4\sin y - 1) = 0$

$\Rightarrow \sin y = 0$ or $\sin y = \frac{1}{4}$

S A

14.5 14.5

T C

$y = 0^\circ, 180^\circ, 360^\circ,$

$14.5^\circ, 165.5^\circ$

8) i) $\log_a 1 - \log_a (a^m)^3$

$= 0 - \log_a a^{3m}$

$= 0 - 3m\log_a a$

$= -3m$

ii) $3^{2x+1} = 1000$

$\log_{10} 3^{2x+1} = \log_{10} 1000$

$(2x+1)\log_{10} 3 = 3$

$2x+1 = \frac{3}{\log_{10} 3}$

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8ii cont)

$$2x = \frac{3}{\log_{10} 3} - 1$$

$$x = \frac{\frac{3}{\log_{10} 3} - 1}{2}$$

$$x = 2.64385$$

$$x = 2.64 \text{ to } 3 \text{ s.f}$$

9)i) $A = \frac{h}{2}\left[y_0 + 2\left[y_1 + y_2 + y_3 + y_4 + y_5\right] + y_6\right]$

$$= \frac{1}{2}\left[0 + 2(4 + 4.9 + 5 + 4.9 + 4) + 0\right]$$

$$= 22.8 \text{ m}^2$$

Volume $= 22.8 \times 50 = 1140 \text{ m}^3$

ii) Left x coord of vehicle $= 3 - 1.8 = 1.2$

A)

When $x = 1.2$

$$y = \frac{5}{81}\left[108 \times 1.2 - 54 \times 1.2^2 + 12 \times 1.2^3 - 1.2^4\right]$$

$$y = 4.352 \text{ m}$$

Vehicle is 4.4m high so 4.352m is too low for vehicle to pass through.

B) $A = \frac{5}{81}\int_0^6 \left(108x - 54x^2 + 12x^3 - x^4\right) dx$

$$= \frac{5}{81}\left[\frac{108x^2}{2} - \frac{54x^3}{3} + \frac{12x^4}{4} - \frac{x^5}{5}\right]_0^6$$

$$= \frac{5}{81}\left[54x^2 - 18x^3 + 3x^4 - \frac{x^5}{5}\right]_0^6$$

$$= \frac{5}{81}\left[\left(54(6)^2 - 18(6)^3 + 3(6)^4 - \frac{6^5}{5}\right) - 0\right]$$

$$= 24 \text{ m}^2$$

Volume $50 \times 24 = 1200 \text{ m}^3$

10) i)

$$y = x^2 - 2x$$

When $x = 5$, $y = 5^2 - 2(5) = 15$

When $x = 5.1$ $y = 5.1^2 - 2(5.1) = 15.81$

Gradient of chord $\frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{15.81 - 15}{5.1 - 5} = 8.1$$

ii)

$$f(x) = x^2 - 2x$$

$$\frac{f(5+h) - f(5)}{h}$$

$$= \frac{(5+h)^2 - 2(5+h) - 15}{h}$$

$$= \frac{25 + 10h + h^2 - 10 - 2h - 15}{h}$$

$$= \frac{8h - h^2}{h}$$

$$= 8 - h$$

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10iii) Letting $h \to 0$ gives gradient of chord which becomes tangent to $f(x)$ at $x=5$. This is $f'(5)$

$f'(5) = 8 - 0 = 8$

iii) Point on $f(x)$ at $x=5$ is $(5, 15)$ gradient 8

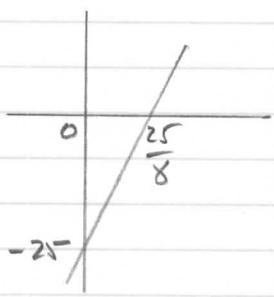
$y - y_1 = m(x - x_1)$

$y - 15 = 8(x-5)$

$y - 15 = 8x - 40$

$y = 8x - 40 + 15$

$y = 8x - 25$



when $y=0$
$8x = 25$
$x = \frac{25}{8}$

Area $= \frac{1}{2} \times 25 \times \frac{25}{8}$

$= 39.0625$ units^2

11) See next two pages.

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11 (i)

$y = a \times 10^{bt}$

$\log_{10} y = \log_{10}(a \times 10^{bt})$

$\log_{10} y = \log_{10} a + \log_{10} 10^{bt}$

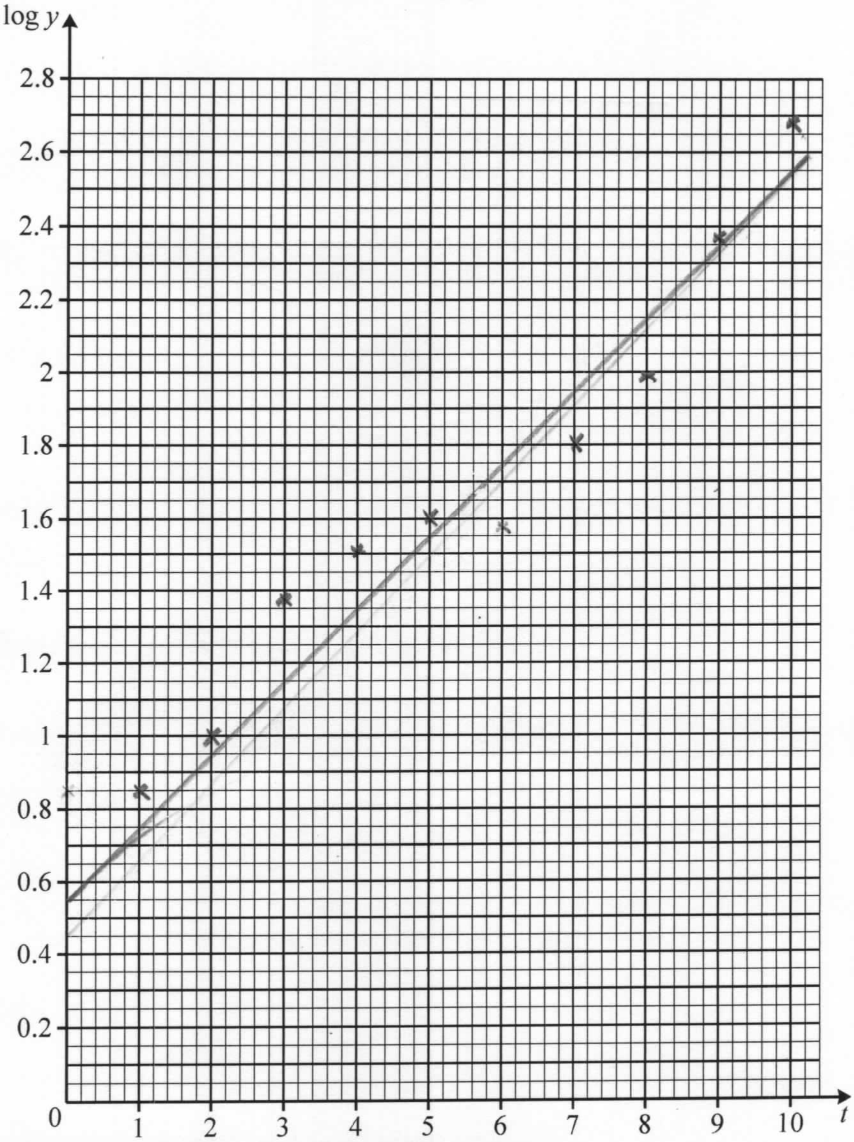
$\log_{10} y = \log_{10} a + bt \log_{10} 10$

$\log_{10} y = \log_{10} a + bt$

Compare with $y = c + mx$ a straight line graph

Intercept $= \log_{10} a$ gradient $= b$

11 (ii) **First complete the copy of the table on the next page.**



The work space for this part is continued on the next page.

11 (ii) continued

t	1	2	3	4	5	6	7	8	9	10
y	7	10	24	32	40	38	63	96	234	480
$\log_{10} y$	0.85	1	1.38	1.51	1.60	1.58	1.80	1.98	2.37	2.68

Intercept $\log_{10} a = 0.55$

$a = 10^{0.55} = 3.55$

Gradient $b = \dfrac{2.55 - 0.55}{10 - 0} = 0.2$

$\therefore \; y = 3.55 \times 10^{0.2t}$

11 (iii)

$y = 921 \times 10^{-0.137w}$

$w = 4 \quad y = 921 \times 10^{-0.137 \times 4} = 260.77$

$y = 261$ is number of viruses