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1) $\sum_{r=3}^{6} \frac{12}{r} = \frac{12}{3} + \frac{12}{4} + \frac{12}{5} + \frac{12}{6}$

$= 4 + 3 + 2.4 + 2$

$= 11.4$

2) $\int \left(3x^5 + 2x^{-\frac{1}{2}}\right) dx$

$= \frac{3x^6}{6} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{x^6}{2} + 4x^{\frac{1}{2}} + c$

3) $\text{Area} \approx \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4) + y_5\right]$

$= \frac{1.5}{2}\left[0.6 + 2(2.3 + 3.1 + 2.8 + 1.8) + 0.7\right]$

$= 15.975 \text{ m}^2$

$\approx 16.0 \text{ m}^2$ to 3 s.f.

4) $y = f(x)$ min point $(3, 5)$

i) $y = 3f(x)$ min point $(3, 15)$

ii) $y = f(2x)$ min point $\left(\frac{3}{2}, 5\right)$

5) GP 2nd $ar = 6$

5th $ar^4 = -48$

$r^3 = \frac{ar^4}{ar} = \frac{-48}{6} = -8$

$\Rightarrow r = \sqrt[3]{-8} = -2$

Sub in 2nd term

$a \times -2 = 6$

$\Rightarrow a = -3$

$\therefore a = -3, \quad r = -2$

10th term $= ar^9$

$= -3 \times (-2)^9$

$= 1536$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{-3((-2)^n - 1)}{-2 - 1}$

$= \frac{-3((-2)^n - 1)}{-3}$

$= (-2)^n - 1$

6) AP 3rd $a + 2d = 24$

10th $a + 9d = 3$

10th − 3rd $7d = -21$

$d = -3$

Sub in 3rd $a - 6 = 24$

$a = 24 + 6$

$a = 30$

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6) cont

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

Require $S_{50} - S_{20}$

$$= \frac{50}{2}\left(60 + 49(-3)\right)$$

$$- \frac{20}{2}\left(60 + 19(-3)\right)$$

$$= 25(-87) - 10(3)$$

$$= -2205$$

7) i)

$$\log_{10} x^5 + 3\log_{10} x^4$$

$$= 5\log_{10} x + 12\log_{10} x$$

$$= 17\log_{10} x$$

ii)

$$\log_a 1 - \log_a a^b$$

$$= 0 - b\log_a a$$

$$= 0 - b$$

$$= -b$$

8)

$$5\sin^2\theta = 5 + \cos\theta$$

$$5(1-\cos^2\theta) = 5 + \cos\theta$$

$$\cancel{5} - 5\cos^2\theta = \cancel{5} + \cos\theta$$

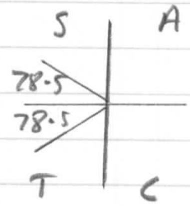
$$0 = 5\cos^2\theta + \cos\theta$$

$$0 = \cos\theta(5\cos\theta + 1)$$

$$\Rightarrow \cos\theta = 0 \text{ or } \cos\theta = -\tfrac{1}{5}$$

$$\cos\theta = 0 \Rightarrow \theta = 90^\circ, 270^\circ$$

$$\cos^{-1}\tfrac{1}{5} = 78.5^\circ$$



$$\cos\theta = -\tfrac{1}{5} \Rightarrow \theta = 101.5, 258.5$$

Solution

$$\theta = 90^\circ, 101.5^\circ, 258.5^\circ, 270^\circ$$

9) Area of sector $= \frac{1}{2}r^2\theta$

$$= \frac{1}{2}r^2\frac{\pi}{6}$$

$$= \frac{\pi r^2}{12}$$

Triangle area $= \frac{1}{2}a^2\sin\frac{\pi}{6}$

$$= \frac{a^2\sin\frac{\pi}{6}}{2}$$

If triangle is half area of sector

$$\frac{a^2\sin\frac{\pi}{6}}{2} = \frac{1}{2} \times \frac{\pi r^2}{12}$$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$$\Rightarrow \frac{a^2}{2} \times \frac{1}{2} = \frac{1}{2} \times \frac{\pi r^2}{12}$$

$\times 4$

$$\Rightarrow a^2 = \frac{\pi r^2}{6}$$

$$\Rightarrow a = r\sqrt{\frac{\pi}{6}}$$

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10 i) A(1,4) B(0,1)

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-4}{1-4} = \frac{x-1}{0-1}$$

$$\frac{y-4}{-3} = \frac{x-1}{-1}$$

$$y-4 = \frac{-3(x-1)}{-1}$$

$$y-4 = 3(x-1)$$

$$y-4 = 3x-3$$

$$y = 3x+1$$

is the line through A and B

Solve

$$\left.\begin{aligned} y &= 3x+1 \\ y &= 4x^2 \end{aligned}\right\}$$

$$4x^2 = 3x+1$$

$$4x^2 - 3x - 1 = 0$$

$$(4x+1)(x-1) = 0$$

$$\Rightarrow \quad x = -\tfrac{1}{4} \quad \text{or} \quad x = 1$$

when $x = -\frac{1}{4}$

$$y = 3(-\tfrac{1}{4})+1 = +\tfrac{1}{4}$$

So intersects at $(-\frac{1}{4}, +\frac{1}{4})$

ii) $y = 4x^2 \Rightarrow \frac{dy}{dx} = 8x$

At A(1,4) $\frac{dy}{dx} = 8(1) = 8$

tgt $y - y_1 = m(x - x_1)$

$$y-4 = 8(x-1)$$

$$y-4 = 8x-8$$

$$y = 8x-4$$

At C($-\frac{1}{4}, \frac{1}{4}$) $\frac{dy}{dx} = 8(-\frac{1}{4}) = -2$

tgt $y - y_1 = m(x - x_1)$

$$y - \tfrac{1}{4} = -2(x - -\tfrac{1}{4})$$

$$y - \tfrac{1}{4} = -2(x + \tfrac{1}{4})$$

$$y - \tfrac{1}{4} = -2x - \tfrac{1}{2}$$

$$y = -2x - \tfrac{1}{2} + \tfrac{1}{4}$$

$$y = -2x - \tfrac{1}{4}$$

iii)

$$\left.\begin{aligned} y &= -2x - \tfrac{1}{4} \quad (1) \\ y &= 8x - 4 \quad (2) \end{aligned}\right\}$$

$$-2x - \tfrac{1}{4} = 8x - 4$$

$$4 - \tfrac{1}{4} = 8x + 2x$$

$$\tfrac{15}{4} = 10x$$

$$\tfrac{15}{40} = x$$

Sub in (2) $y = 8(\frac{15}{40}) - 4$

$y = -1$ at D

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11) $y = x^3 - 3x^2 - x + 3$

i) $\int_1^3 (x^3 - 3x^2 - x + 3)\,dx$

$= \left[\frac{x^4}{4} - x^3 - \frac{x^2}{2} + 3x\right]_1^3$

$= \left(\frac{3^4}{4} - 3^3 - \frac{3^2}{2} + 3(3)\right)$

$- \left(\frac{1^4}{4} - 1^3 - \frac{1^2}{2} + 3(1)\right)$

$= \left(\frac{81}{4} - 27 - \frac{9}{2} + 9\right)$

$- \left(\frac{1}{4} - 1 - \frac{1}{2} + 3\right)$

$= \left(-\frac{9}{4}\right) - \left(\frac{7}{4}\right)$

$= -\frac{16}{4} = -4$

Area between curve and x-axis between $x = 1$ and $x = 3$

Area is 4 units^2

Minus sign indicates area is below x-axis

ii) $\frac{dy}{dx} = 3x^2 - 6x - 1$

At t.p $\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 6x - 1 = 0$

$x = \frac{6 \pm \sqrt{36 + 4 \times 3 \times 1}}{6}$

$x = \frac{6 \pm \sqrt{48}}{6}$

$x = \frac{6 \pm \sqrt{16 \times 3}}{6}$

$x = \frac{6 \pm 4\sqrt{3}}{6}$

$x = \frac{3 \pm 2\sqrt{3}}{3}$

Decreasing function between the maximum and minimum points

That is for

$\frac{3 - 2\sqrt{3}}{3} < x < \frac{3 + 2\sqrt{3}}{3}$

(5)

12 (i)

Decrease in population between 1980 and 2005

$= 25000 - 12000 = 13000$

Percentage decrease $= \frac{13000}{25000} \times 100\%$

$= 52\%$ in 25 years

so red alert

12 (ii)

$P = a \times 10^{-kt}$

$\log_{10} P = \log_{10}(a \times 10^{-kt})$

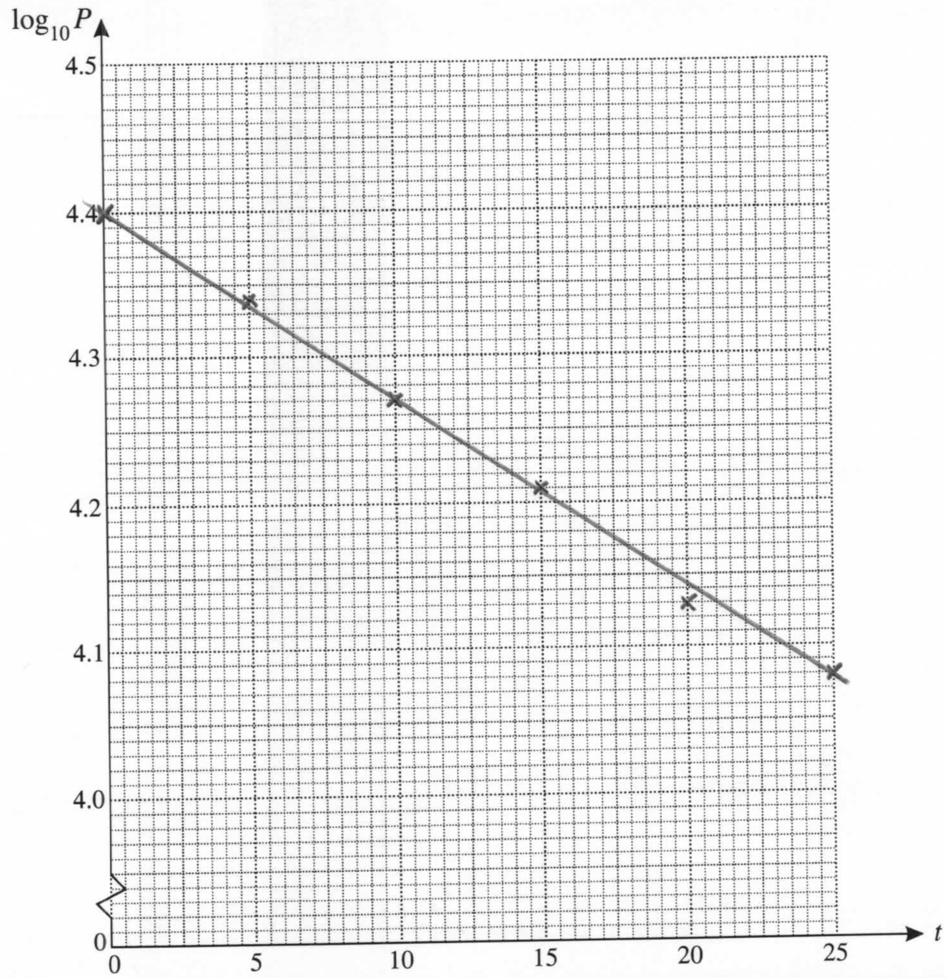
$\log_{10} P = \log_{10} a + \log_{10} 10^{-kt}$

$\log_{10} P = \log_{10} a - kt$

12 (iii)

Year	1980	1985	1990	1995	2000	2005
Population (P)	25 000	22 000	18 750	16 250	13 500	12 000
Years after 1980 (t)	0	5	10	15	20	25
$\log_{10} P$	4.40	4.34	4.27	4.21	4.13	4.08

12 (iii) **(continued)**



12 (iv)

$\log_{10} a$ = vertical intercept = 4.4

$\Rightarrow a = 10^{4.4} = 25119$

$-k$ is gradient of graph $= \dfrac{4.4 - 4.08}{0 - 25} = -0.0128$

$\therefore a = 25119, \; k = 0.0128$

$$P = 25119 \times 10^{-0.0128t}$$

12v) $t = 35$ (in 2015) $P = 25119 \times 10^{-0.0128 \times 35} = 8954$

As % of 1990 population $\dfrac{8954}{18750} = 47.8\%$ so dropped 52.2%

Yes still on red alert